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Feedback on the development of measurement concepts in the draft mathematics and statistics curriculum

The development of an understanding of measurement is a critical one that crosses all of the new curriculum strands. It is one that needs to be developed and explained to teachers in some detail (for reasons outlined below) so needs second tier support, ands better integration in the A/Os.

The existing curriculum, while following common developmental progressions (for example see Zevenbergen , Dole & Wright (2004), has not led to a good understanding of measurement in our students. In terms of learning about length, the simplest measurement case, many seem to have moved from 'measuring by counting (hands etc)' at level 1 to 'reading the scale on a ruler' at level 2, which is what the A/Os effectively say to do. A ruler, like most other measurement devices, is a sophisticated tool that uses a variety of scales. To cope with this 'leap' many students seem to have learned the 'rule' that 'the little marks between the centimetres are tenths'. Fully one third of our students at year 8 have no strategy for working out the value of an interval if it is not in tenths (see NEMP 2005). This approach has resulted in problems in measurement, the measure construct of fractions, and in statistical and algebraic graphing. These are all well-documented in the research literature in these fields, so should not need to be elaborated upon here.

Proposed changes to the measurement progression

Level 1

- The second tier material should stress the need to start the measurement process by identifying the attribute. Currently, many lower ability secondary students are trying to use formulae to find areas and still do not have a good concept of what 'area' is...
- The A/O here mentions order and compare ... by direct comparison. Many progressions for measurement include indirect comparison here. Direct and indirect comparison also tends to be 'number-free', where the language of the attribute is developed. This should be spelt out in the second tier material.
- The use of 'counting whole numbers of units' implies that a unit needs to be developed. This is usually tied to the development of non-standard units, and the counting of non-standard units (with the unit being appropriate to the attribute). This needs to be made more explicit as it is an important step in the development of measurement both historically and in the mathematics classroom.

Level 2

- The emphasis on 'create and use appropriate units' seems to indicate that both non-standard and standard units are being developed for level 2. Rewording is needed here given that 'units' are already developed in level 1. 'Create and use appropriate *standard* units and devices...' would cover the change. This would also highlight the need to move from counting strategies to measurement strategies – considering the role of zero, the iteration of a unit to create a scale, the use of measurement conventions for locating numbers, whether to count the marks or the spaces, and the concept that the 'next' is where 'one' is found. However, all of this would need to be spelt out in the second tier material if teachers are expected to cover the significant points here.
- The emphasis on creating a scale is a good change. This needs to be done before a ruler or other standard measurement instruments are met. The emphasis on appropriate units is also important.
- Reunitising should also be included at this stage. Reunitising is a common process that is used by young students to answer questions which require greater accuracy than whole numbers allow. They are familiar with reunitising from using money and when sharing. Reunitising is also a common measurement strategy. Measuring as '3cm and 4mm' would also allow students

to measure with more accuracy and introduce the concept that units can be subdivided, and that smaller units need to be used to measure with greater accuracy – all without the use of decimals.

- The concept of a half should also be introduced here, given that many students 'treat it as an honorary whole number'. This also introduces students to the idea that intervals/units can be partitioned and that fractions can be found between whole numbers.
- Scales should be restricted to unit scales.
- Three types of ruler are consistent with the above. A centimetre ruler that is blank between the whole centimetres, a ruler marked in millimetres, and a centimetre ruler with millimetres shown (for reunitising). All of these should start with a zero (as opposed to current rulers that have cm). Use of these rulers needs to be spelt out in the second tier material.

Level 3

- Standardised (linear) scales are distinctly different from unit simple scales, so the distinction between level 2 and level 3 is good to see. Standard scales often involve multiples. For example, a weighing scale may have a scale marked in intervals of 100grams, with 3 additional marks between each number. Students really need to be multiplicative to work out that each division represents 25 grams.
- Given that simple decimals are introduced at level 3 in number, and students are expected to use simple multiplicative strategies, I can see no reason why the concept of tenths cannot be introduced in relation to measurement here. The development of decimal place value should be able to be linked to conversion factors between units in the metric system.
- Students who can work out the value of an interval on a whole number scale should be able to transfer this skill to the fraction (or decimal) context – as adults do.

Level 4

- 'Real' measurement often involves the use of a measurement instrument with a variety of scales, for example a weighing scale in pounds and in kilograms. The recognition of these 'alternative realities', appropriate use of these different scales and conversion factors between them are again a step up from using a single scale, so should be detailed at this level.

Other factors in the development of measurement.

1) The measurement progression outlined above has implications for the other strands. For example, statistics. The development of the concept of probability (the measurement of chance) can be mapped onto this progression, as can most statistical surveys – try it by investigating favourite car colours – though you need a large number of cars to get to the point that a non-unit vertical scale needs to be developed. This implies that the content of the statistics strand needs to take into account the development of measurement concepts where graphing is required.

   Level 1 – Using the statistical enquiry cycle in investigations involving counting based on categoric data. Use of counting conventions and the need for a common baseline.

   Level 2 – The statistical enquiry cycle in investigations that involve the introduction of measurement conventions for scales, the role of zero on the scale, equal spacing of numbers, how to deal with 'missing' numbers... Graphs should use unit scales so collecting small amounts of data should be a feature of the investigations.

   Level 3 – The statistical enquiry cycle involving larger data sets and working with more sophisticated scales, including multiples. For example, reading numbers from a graph where the reading is *between* the numbers marked on the scale. (This skill is required when reading most 'real' graphs, but has not been systematically taught in recent years.)

Level 4 – The statistical enquiry cycle involving measurement data and grouped discrete data. This would then allow the differing conventions for discrete, grouped discrete and continuous data to be highlighted.

Note that an emphasis on developing the notion of the attribute being studied is a powerful way of getting students to focus on how to compare and display that attribute. For example, in a survey on heights, some students, even up to year 13, persist on graphing unnecessary information – such as plotting each students' name and their height rather than creating a display to show the distribution of heights.

2) In the development of graphing, the introduction of discrete numeric data often includes the ability to read and plot data presented in table form, as well as understanding that the horizontal axis of a graph is really a scale, rather than the baseline found when using categoric data. This is poorly understood by many students, and needs to be stressed in the support material. (My research, has found that a significant number of year 7 and 8 students still treat the horizontal axis as a baseline and the numbers as categories. Some other studies have similar findings.) Some year 7 and 8 students also do not realise that the integrity of the data pairs needs to be maintained, so they do things like plot one set of the data twice. My conclusion is that algebraic graphs need to be introduced before discrete numeric data is touched, as this can be done in a 'clean' environment without 'contextual pollution'. While the CL2 equations A/O allows for graphs to be introduced, this word should be introduced to the A/O. This introduction of graphs with unit scales should be highlighted in the support material as this would be a change in practice.

3) Again to be consistent with the development of measurement, 'graphs' (with non-unit scales) should be introduced to the level 3 equations and expressions A/O, This should involve tasks like 'plotting points involving numbers like 3, 5, 8, 37 on a scale marked in multiples of five', and reading points from such scales. One way of doing this is to subdivide an interval into 5 equal parts, which ties to understanding the measure construct of fractions. Such learning has traditionally been left to chance, rather than has been something explicitly taught, and is not well understood. Problems should be based on reading graphs and plotting tables of data where the data set involves either a linear or a non-linear pattern, or no particular pattern at all. This practice reduces the belief that 'all algebraic graphs draw straight lines' and would then allow level 4 to focus on the specific study of linear relationships.

4) An explanation of the various fraction constructs should be added to the support material for number, along with when they should be introduced. In relation to the development of measurement concepts, the measure construct of fractions needs to be specifically mentioned. 'In the good old days' developing an interval concept of halves, quarters and eighths came from the imperial ruler, and was early learning – before fifths and tenths were introduced. This can tie to level 2, is consistent with halves being a honorary whole number and also consistent with number development. Dividing an interval into *n* equal pieces (and that they are called *n*ths) can be held over to level 3 as this has been much harder for the students I have worked with, so is probably more suited there.

Using number lines in this way is another skill that has been overlooked in recent years, but is important for accurate measurement and graph work. 'Oldies' got regular doses from working on imperial rulers, number lines and slide rules. Now students are expected to pick up the skills but are never officially taught them.

5) When operating on scales, my study has identified that students (and adults) strategise when working with non-unit scales. Some of these strategies can be classed as counting based, while others are more related to addition or multiplication. That such a progression in strategy sophistication exists should not be a surprise given that student understanding of number can be placed in such a progression. Restricting level 2 to unit scales should reduce the over-generalisation of the lower level 'counting' strategies that I have observed. For example – assuming that every mark stands for one, or always counting in tenths when

working between whole numbers. This also allows scales that require multiplicative strategies (most non-unit scales) to be held until curriculum level 3, the level where students can be expected to be multiplicative thinkers. The one exception to this process could be the use of a scale in twos, where 'halving' the interval would identify the missing unit. This would also be consistent with the measurement progression and the introduction of one half of an interval at level 2.

Note: I believe that second tier material - along the lines of the old 'blue' statistics and 'yellow' calculus support books needs to be provided for teachers at all levels of the curriculum.

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References

Zevenbergen, R., Dole, S. and Wright, R. (2004). *Teaching mathematics in primary schools.* Crows Nest, NSW: Allen and Unwin.

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Dear Michael Drake

Thank you for the submission on the *New Zealand Curriculum: Draft for Consultation 2006*. Your views will be included in the feedback we are gathering to guide the writing of the final New Zealand Curriculum document, due to be released next year.

Yours sincerely

HL

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