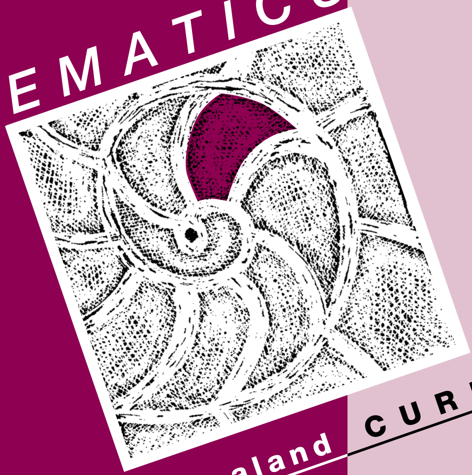
MATHEMATICS in the New Zealand CURRICULUM



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Contents

Foreword

This curriculum statement replaces the syllabuses Mathematics: Junior Classes to Standard 4 and Mathematics: Forms 1 to 4. The statement specifies the mathematics learning area described in The New Zealand Curriculum Framework and provides the basis for mathematics programmes in schools from Year 1 to Year 13, that is, from Junior classes to Form 2 in primary schools and from Form 3 to Form 7 in secondary schools.

The Minister of Education initiated a redesign of the mathematics curriculum in 1991, as part of a broad initiative aimed at improving achievement levels in the essential subjects of the curriculum. The review and writing were undertaken by a small project team under the general guidance of a mathematics advisory committee. A draft statement was prepared and circulated to schools and interested groups for comment and discussion. The views of all those who responded have been taken into account in preparing the final curriculum statement.

There are three major directions in this mathematics curriculum statement. It gives special emphasis to continuity and progression in learning in mathematics by specifying clear learning goals expressed as achievement objectives at eight levels through all of the years of schooling. The statement focuses on the importance of diagnostic and formative assessment to enhance the teaching and learning process for all students. Finally, the curriculum statement stresses the need for mathematics to be taught and learned within the context of problems which are meaningful to students and which lead to understanding of the way mathematics is applied in the world beyond school.

The Ministry is grateful to all who have contributed to the development of this curriculum statement, especially the contractor and the writers, the contract review committee, and the members of the policy advisory group all of whom gave freely of their time and experience.

Maris O'Rourke, PhD
Secretary for Education

Introduction

Mathematics is a coherent, consistent, and growing body of concepts which makes use of specific language and skills to model, analyse, and interpret the world. Mathematics provides a means of communication which is powerful, concise, and unambiguous.

As a human endeavour, mathematics involves creativity and imagination in the discovery of patterns of shape and number, the perceiving of relationships, the making of models, the interpretation of data, and the communication of emerging ideas and concepts.

The New Zealand Curriculum Framework includes mathematics as one of seven essential areas of learning. Mathematical understanding and skills contribute to people's sense of self-worth and ability to control aspects of their lives. Everyone needs to develop mathematical concepts and skills to help them understand and play a responsible role in our democratic society. Mathematics education aims to provide students with those skills and understandings.

The New Zealand Curriculum Framework also asserts the importance of eight essential sets of skills: communication skills; numeracy skills; information skills; problem-solving skills; self-management and competitive skills; social and co-operative skills; physical skills; work and study skills.

The need for people to be numerate, that is, to be able to calculate, estimate, and use measuring instruments, has always been identified as a key outcome for education. Mathematics education aims to contribute to the development of the broad range of numeracy skills.

In an increasingly technological age, the need for innovation, and problem-solving and decision-making skills, has been stressed in many reports on the necessary outcomes for education in New Zealand. Mathematics education provides the opportunity for students to develop these skills, and encourages them to become innovative and flexible problem solvers.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations, thereby contributing to the development of many social and co-operative skills.

Increasingly, information is communicated through the use of data graphics. The communication of information through graphics is particularly common in the mass media. It is important that people can draw sensible conclusions from charts, tables, and graphs of various kinds. At the same time, increasing numbers of occupations demand the ability to collect data, to understand and use information technology for the organisation and interpretation of data, and to present reports and summaries. Mathematics education gives young people the opportunity to develop information skills through learning and practising data handling and data interpretation.

General Aims of Mathematics Education

Mathematics education aims to:

- ❏ help students to develop a belief in the value of mathematics and its usefulness to them, to nurture confidence in their own mathematical ability, to foster a sense of personal achievement, and to encourage a continuing and creative interest in mathematics;
- ❏ develop in students the skills, concepts, understandings, and attitudes which will enable them to cope confidently with the mathematics of everyday life;
- ❏ help students to develop a variety of approaches to solving problems involving mathematics, and to develop the ability to think and reason logically;
- ❏ help students to achieve the mathematical and statistical literacy needed in a society which is technologically oriented and information rich;
- ❏ provide students with the mathematical tools, skills, understandings, and attitudes they will require in the world of work;
- ❏ provide a foundation for those students who may continue studies in mathematics or other learning areas where mathematical concepts are central;
- ❏ help to foster and develop mathematical talent.

Achievement Aims of the Mathematics Curriculum

Mathematical Processes

The mathematics curriculum intended by this statement will provide opportunities for students to:

- develop flexibility and creativity in applying mathematical ideas and techniques to unfamiliar problems arising in everyday life, and develop the ability to reflect critically on the methods they have chosen;
- become effective participants in problem-solving teams, learning to express ideas, and to listen and respond to the ideas of others;
- develop the skills of presentation and critical appraisal of a mathematical argument or calculation, use mathematics to explore and conjecture, and learn from mistakes as well as successes;
- develop the characteristics of logical and systematic thinking, and apply these in mathematical and other contexts, including other subjects of the curriculum;
- become confident and competent users of information technology in mathematical contexts;
- develop the skills and confidence to use their own language, and the language of mathematics, to express mathematical ideas;
- develop the knowledge and skills to interpret written presentations of mathematics.

Number

The mathematics curriculum intended by this statement will provide opportunities for students to:

- develop an understanding of numbers, the ways they are represented, and the quantities for which they stand;
- develop accuracy, efficiency, and confidence in calculating — mentally, on paper, and with a calculator;
- develop the ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements.

Measurement

The mathematics curriculum intended by this statement will provide opportunities for students to:

- develop knowledge and understanding of systems of measurement and their use and interpretation;
- develop confidence and competence in using instruments and measuring devices;
- predict and calculate the effects of changes in variables and rate of change of variables on systems representable by simple mathematical models.

Geometry

The mathematics curriculum intended by this statement will provide opportunities for students to:

- gain a knowledge of geometrical relations in two and three dimensions, and recognise and appreciate their occurrence in the environment;
- develop spatial awareness and the ability to recognise and make use of the geometrical properties and symmetries of everyday objects;
- develop the ability to use geometrical models as aids to solving practical problems in time and space.

Algebra

The mathematics curriculum intended by this statement will provide opportunities for students to:

- recognise patterns and relationships in mathematics and the real world, and be able to generalise from these;
- develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalisations;
- use algebraic expressions confidently to solve practical problems.

Statistics

The mathematics curriculum intended by this statement will provide opportunities for students to:

- recognise appropriate statistical data for collection, and develop the skills of collecting, organising, and analysing data, and presenting reports and summaries;
- interpret data presented in charts, tables, and graphs of various kinds;
- develop the ability to estimate probabilities and to use probabilities for prediction.

Development of Essential Skills through Mathematics

The key outcome of mathematics education is the development of the ability to apply certain of the essential skills described in The New Zealand Curriculum Framework — communication skills; numeracy skills; information skills; problem-solving skills; social and co-operative skills; and work and study skills. The mathematical processes identified in this curriculum statement are the expression of these essential skills in mathematical contexts. This curriculum statement, therefore, suggests approaches to teaching, learning, and assessment which will give students the maximum possible opportunity to develop the essential skills.

Approaches to Teaching and Learning in Mathematics

Problem-solving Approach

A balanced mathematical programme includes concept learning, developing and maintaining skills, and learning to tackle applications. These should be taught in such a way that students develop the ability to think mathematically.

Students learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them. Thus, mathematics is best taught by helping students to solve problems drawn from their own experience.

Real-life problems are not always closed, nor do they necessarily have only one solution. Determining the best approximation to a solution, and finding the optimum way of solving a problem when several approaches are possible, are skills frequently required in the workplace. Students need frequent opportunities to work with open-ended problems. The solutions to problems which are worth solving seldom involve only one item of mathematical understanding or only one skill. Rather than remembering the single correct method, problem solving requires students to search the information for clues and to make connections to the various pieces of mathematics and other knowlege and skills which they have learned. Such problems encourage thinking rather than mere recall.

Closed problems, which follow a well-known pattern of solution, develop only a limited range of skills. They encourage memorisation of routine methods rather than consideration and experimentation. While fluency with basic techniques is very important, such routines only become useful tools when students can apply them to realistic problems.

The characteristics of good problem-solving techniques include both convergent and divergent approaches. These include the systematic collection of data or evidence, experimentation (trial and error followed by improvement), flexibility and creativity, and reflection — that is, thinking about the process that has been followed and evaluating it critically.

Teachers can create opportunities for students to develop these characteristics by encouraging them to practise and learn such simple strategies as guessing and checking, drawing a diagram, making lists, looking for patterns, classifying, substituting, re-arranging, putting observations into words, making predictions, and developing proofs.

Learning to communicate about and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically. Critical reflection may be developed by encouraging students to share ideas, to use their own words to explain their ideas, and to record their thinking in a variety of ways, for example, through words, symbols, diagrams, and models.

The chance to look for problems as well as to solve them, to create and to produce rather than reproduce what already exists, is important for all students. Creativity in problem solving is recognised as one of the basic traits that must be developed if outstanding achievement is to result, and it plays a major role in innovation, invention, and scientific discovery.

Catering for Individual Needs

It is a principle of the New Zealand Curriculum Framework that all students should be enabled to achieve personal standards of excellence and that all students have a right to the opportunity to achieve to the maximum of their potential. It is axiomatic in this curriculum statement that mathematics is for all students, regardless of ability, background, gender, or ethnicity.

Students of lower ability need to have the opportunity to experience a range of mathematics which is appropriate to their age level, interests, and capabilities. Equally, students with exceptional ability in mathematics must be extended and not simply expected to repeat different permutations of work they have clearly mastered.

As new experiences cause students to refine their existing knowledge and ideas, so they construct new knowledge. The extent to which teachers are able to facilitate this process significantly affects how well students learn. It is important that students are given explicit opportunities to relate their new learning to knowledge and skills which they have developed in the past. Factors such as out-of-school experience and language have profound effects on the way students learn mathematics.

In many cases in the past, students have failed to reach their potential because they have not seen the applicability of mathematics to their lives and because they were not encouraged to connect new mathematical concepts and skills to experiences, knowledge, and skills which they already had. This has been particularly true for many girls, and for many Maori students, for whom the contexts in which mathematics was presented were irrelevant and inappropriate. These students have developed deeply entrenched negative attitudes towards mathematics as a result.

An awareness of these issues has led to improved access for girls to mathematics, but the participation rate of female students in mathematics continues to be lower than that of male students at senior school level and beyond. This limits later opportunities for girls and women.

The suggested learning experiences in this document include strategies that utilise the strengths and interests that girls bring to mathematics. Techniques that help to involve girls actively in the subject include setting mathematics in relevant social contexts, assigning co-operative learning tasks, and providing opportunities for extended investigations.

The suggestions also describe experiences which will help girls develop greater confidence in their mathematical ability. Girls' early success in routine mathematical operations needs to be accompanied by experiences which will help them develop confidence in the skills that are essential in other areas of mathematics. Girls need to be encouraged to participate in mathematical activities involving, for example, estimation, construction, and problems where there are any number of methods and where there is no obvious "right answer".

It is particularly important that mathematical learning experiences for Maori students acknowledge the background experiences which have led to the formation of ideas and skills which those students already have. Maori students will be helped to achieve if teachers acknowledge and value those ideas and experiences.

Traditional time-constrained pencil and paper tests have proved unreliable indicators of Maori achievement in the past. Among the sample assessment activities, there are procedures suggested which may be more appropriate for assessing Maori students. In selecting assessment procedures, teachers should endeavour to ensure that all of the desired objectives are evaluated and that the procedures which are selected are culturally appropriate.

The development of more positive attitudes to mathematics and a greater appreciation of its usefulness is the key to improving participation rates for all students.

Use of Resources

Apparatus

The importance of the use of apparatus to help students form mathematical concepts is well established. Using apparatus provides a foundation of practical experience on which students can build abstract ideas. It encourages them to be inventive, helps to develop their confidence, and encourages independence.

Junior school teachers are used to choosing an appropriate range of apparatus to focus students' thinking on the concept to be developed and modifying the apparatus as the learner's understanding grows. Teachers know that students are capable of solving quite difficult problems when they are free to use concrete apparatus to help them think the problems through. Such an approach is equally valid with older students and should be used wherever possible.

At all levels, students should be introduced to new ideas by having their attention drawn to examples occurring in their natural environment, and then by modelling them with apparatus. For example, a child's concept of "four" could be enriched by discussing the number of wheels on a car, legs on a table, or edges on a piece of paper. The child could then be encouraged to explore the idea further, using materials with which to make their own models of "four". Similarly, secondary students could be focused on the concept of "rate of change" by discussing, for example, that younger people grow faster than older people, or by discussing the slope changes on nearby hills. Students could then model uniform and non-uniform rate situations, using apparatus such as sand running through an egg-timer or a ball rolling down a smooth slope.

Textbooks

Many textbooks contain material to provide students with practice and enrichment. Increasing numbers of books contain excellent ideas for problem-solving situations which develop mathematical skills and understandings. However, teachers must realise that there are dangers in adhering too closely to any particular textbook. Many texts contain material not included in this curriculum statement, or have emphases which are different from those advocated for New Zealand. In any event, teachers should continually re-evaluate the texts they are using in the light of the particular needs of their students.

Technology

This curriculum statement assumes that both calculators and computers will be available and used in the teaching and learning of mathematics at all levels. Instruction in the correct and appropriate use of calculators is particularly important.

Calculators, graphics calculators, and computers are learning tools which students can use to discover and reinforce new ideas. Calculators are powerful tools for helping students to discover numerical facts and patterns, and helping them to make generalisations about, for example, repeated operations. Graphics calculators, and computer software such as graphing packages and spreadsheets, are tools which enable students to concentrate on mathematical ideas rather than on routine mechanical manipulation, which often intrudes on the real point of particular learning situations. Computer programs, such as Logo, provide excellent environments for mathematical experimentation and open-ended problem solving.

Assessment and Evaluation in Mathematics

Evaluation of students' achievement is an essential part of mathematics education.

Monitoring and evaluation are necessary to assess students' readiness for new learning, to give teachers feedback on the success of their methods and approaches, and to assist planning for new learning.

Evaluation includes diagnostic assessment procedures which enable teachers to discover difficulties that individual students may be having. Appropriate diagnostic assessment may reveal that the reason for a particular student's lack of progress is a lack of understanding achieved at some earlier time, and the difficulty may be relatively easily addressed. Diagnosis may also reveal that the student is very talented and is simply bored by lack of stimulation. Diagnostic assessments enable teachers to plan further learning activities specifically designed to meet the learning needs of individual students. Worthwhile diagnosis is very often carried out by simple question and answer interaction in the classroom.

Assessment should focus both on what students know and can do, and on how they think about mathematics. It should involve a broad range of mathematical tasks and problems and require the application of a number of mathematical ideas. Skills assessed should include the ability to communicate findings, to present an argument, and to exploit an intuitive approach to a problem.

Assessment should, as far as possible, be integral to the normal teaching and learning programme. Continuing assessment as part of the teaching and learning programme increases the range and quality of assessment which can be carried out for good diagnosis, and avoids the artificial intrusion on learning and teaching time which is associated with separate assessment sessions. Assessment should involve multiple techniques including written, oral, and demonstration formats. Group and team activities should also be assessed.

Teachers should avoid carrying out only tests which focus on a narrow range of skills such as the correct application of standard algorithms. While such skills are important, a consequence of a narrow assessment regime which isolates discrete skills or knowledge is that students tend to learn in that way. Mathematics becomes for them a set of separate skills and concepts with little obvious connection to other aspects of learning or to their world.

Assessment should also be undertaken to provide students and their parents with an indication of a student's progress. In summarising the results of evaluations of students' achievement, teachers should report what students have been working on, what they have achieved, and how well they have achieved it. A grade, level, or mark alone is insufficient.

Format and Presentation of this Document

Strands

There are six main achievement aims of the mathematics curriculum. Accordingly, the curriculum statement is presented in six "strands" each of which reflects a particular aim of the curriculum. The strands are headed:

mathematical processes
number
measurement
geometry
algebra
statistics

This division is a convenient way of categorising the outcomes for mathematics education in schools. It emphasises that there are a number of aspects which are all equally important. The division does not mean that mathematics is expected to be learned in discrete "packages". On the contrary, the mathematical processes strand is deliberately intended to encourage teachers and students to make connections between the other strands wherever possible.

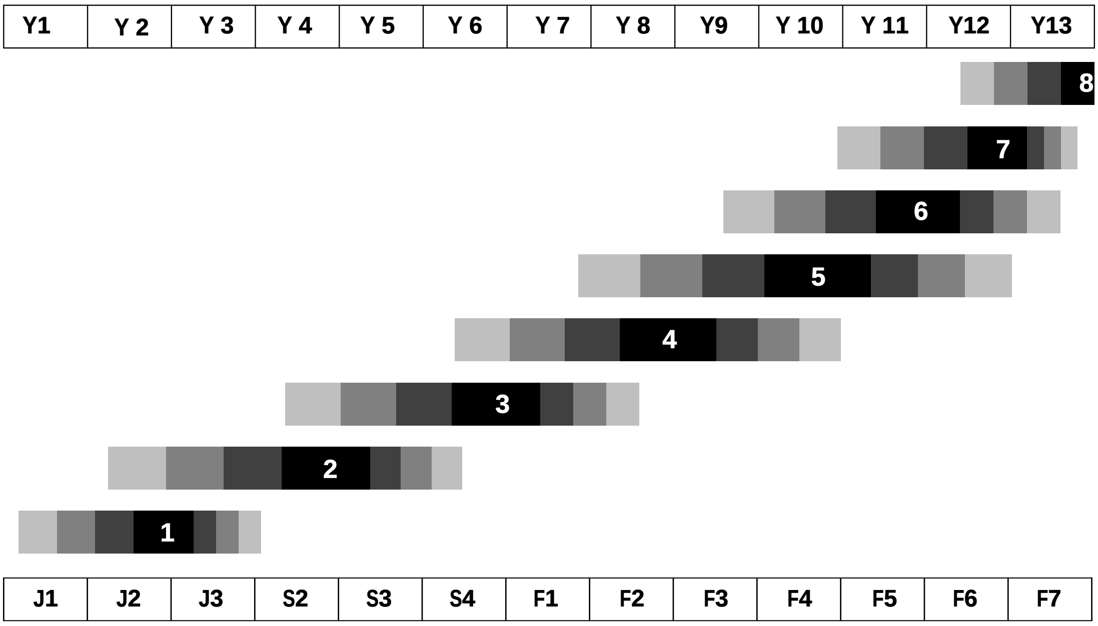
Achievement Objectives by Levels

Each strand, other than "number", is divided into eight levels describing the development of the mathematics curriculum from junior primary school (Year 1) to seventh form (Year 13).

A number of achievement objectives are described in each strand, and at each level. The objectives define what students should be able to achieve after appropriate learning experiences in mathematics. They define the progression of learning outcomes which is the core of this curriculum statement in mathematics.

At each level the objectives are quite broad. It is expected that, in assessing students' progress, teachers will make judgments as to an individual's degree of achievement of particular objectives, and will include commentary on that degree of achievement when reporting to parents.

The number of levels has been chosen for consistency with the New Zealand Curriculum Framework. The division of the school mathematics curriculum into eight levels does not mean that there are eight well-identified stages, which learners pass through in the development of mathematical understanding. However, it is accepted that some concepts are better introduced to older students, and that the effective learning of some ideas depends on a prior understanding of other ideas. The judgment of experienced teachers as to what students can do at various ages has been combined with recent research into mathematical learning to place material into levels. The general relationship between the levels and years at school is described in the diagram on the following page.



This scheme explicitly recognises that each learner is an individual whose learning development and rate of progress is different from others. Different students will be ready for particular mathematical content and experiences at different times. It is not expected that all students of the same age will be achieving at the same level at the same time, nor that an individual student will necessarily be achieving at the same level in all strands of the mathematics curriculum.

The levels are not meant to be interpreted as the rungs of a ladder which is to be climbed as quickly as possible. Nor are they meant to be interpreted as hurdles over which each student must pass before moving to any new work. Rather, they are meant to focus the mathematics programmes of schools in a consistent way. They provide a basis for reporting students' achievements to parents in a way that is clear and demonstrates progression in learning.

The number strand is divided into six levels only. Most of the important achievement objectives for number are to be met in the early years of schooling. In later years, the classification of mathematics into strands is somewhat arbitrary. Some work, for example, numerical analysis, calculus, and complex numbers, which might have been classified under "number", has more usefully been placed in other strands, for example, algebra.

Suggested Learning Experiences

In each strand, and at each level, a range of suggested learning experiences is suggested. The activities and experiences which are included are drawn from the best of contemporary teaching practice, and are intended to help students meet the aims and achievement objectives of the mathematics curriculum.

There is not necessarily an exact match between the suggested learning experiences and the achievement objectives at each level. In some cases, this is because the learning experiences described contribute to concepts and skills which will take considerable time to develop, and appropriate achievement objectives are not described until later levels. At the same time, all of the suggested learning experiences contribute to the development of the broader aims of the curriculum and thus, for some, there may not be specifically associated achievement objectives.

The suggested learning experiences are, nevertheless, pointers only. It is not intended that the activities described in this document should limit the way teachers choose to teach mathematics. Indeed, teachers are encouraged to use their own judgment in designing courses to provide their students with mathematical experiences which will enable the students to achieve the broader aims and achievement objectives of the curriculum. Teachers in, for example, bilingual schools or Kura Kaupapa Maori may choose to offer mathematics in contexts which provide quite different activities and experiences.

The suggested learning experiences are carefully worded in active terms. This is to emphasise that mathematics is most effectively learned through students' active participation in mathematical situations, rather than through passive acceptance and repetition of knowledge.

Sample Assessment Activities

Traditionally, assessment in mathematics has been focused on a quite narrow range of procedures. Procedures such as pencil and paper tests of algorithmic skills do not always reveal students' difficulties, nor do they allow assessment of the full range of students' achievements.

This curriculum statement provides, at each level in each strand, examples of activities in which teachers might engage students to assess their current level of achievement. An assessment programme modelled on these examples will help teachers to plan the next stages of learning for their students.

The models illustrate tasks that can be used to assess a full range of accomplishments, including, for example, the ability to collect and summarise data, the ability to communicate findings, the ability to present an argument, and the ability to exploit an intuitive approach to a problem. The suggestions include multiple assessment techniques including written, oral, and demonstration formats, which should be used in addition to more traditional tests and assignments. Suggestions for group and team assessments are included.

The activities illustrate assessment techniques which are not disruptive to normal classroom activities — they could be carried out as an integral part of the teaching programme rather than at times specifically set aside for "tests". Assessment and evaluation strategies of this kind require teachers to make systematic observations of students at work, and to record their observations carefully.

As each achievement objective in this statement is capable of being achieved at a range of standards, teachers should choose assessment and reporting methods which reveal a student's degree of attainment of the objectives.

The few assessment activities suggested in the statement are exemplars which teachers could imitate in developing their own assessment programme. They provide for teachers a selection of activities which allow for observations of various manifestations of students' achievement. While it is expected that teachers will use the tasks described as models for developing their own assessment tasks and procedures, they are free to use different assessment methods if they wish, and are encouraged to do so. For example, teachers in bilingual schools or Kura Kaupapa Maori may decide to use alternative methods to assess students' progress towards the achievement objectives.

The examples do not cover all of the objectives of the curriculum. A comprehensive assessment programme remains the responsibility of the teacher.*

Development Band

Some students develop faster in all aspects of mathematics than most of their peer group. Other students reach a particular achievement level in one strand or topic sooner or faster than most of their peer group without necessarily being equally competent in all other strands at the same level. A levels structure may be thought to imply that faster students should automatically be accelerated to the next level. This is not necessarily so, nor is it the aim of this curriculum. Teachers should carefully appraise the experience and needs of students before deciding to move them to the next level.

It is very important, however, that students do not have their mathematical development inhibited by, for example, repeating work which they have clearly mastered.

The mathematics contained in the suggested learning experiences at any level is only a subset of the mathematics which students could possibly learn. Faster students can be extended in their mathematical experience without necessarily accelerating them to a higher level, which for many students may itself limit the extent of their learning.

The intention of the development band is to encourage teachers to offer broader, richer, and more challenging mathematical experiences to faster students. Work from the development band should allow better students to investigate whole new topics which would not otherwise be studied and to work at a higher conceptual level. Talented students should have their interest in mathematical ideas further stimulated and their understanding of the nature of mathematics deepened. Teaching approaches which may build on the interest of students include: allowing students themselves to select the topic or content they wish to pursue and to set their own goals; allowing the opportunity for individual and independent study, perhaps using a contract plan; and encouraging access to a broader range of higher level resources.

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* Each example is accompanied by one or more objectives which could be assessed by the activity described. Because the assessable objectives may come from any strand or lower level their origin is indicated. For example, (N5) signifies "Number, Level 5". Equally, the entire objective may not be applicable for each example. In such cases, an ellipsis (…) replaces the inappropriate text.

The development band must not be considered as an optional extra or simply a reward for good work. Students have a right to the opportunity to extend their mathematical knowledge and power. Accordingly, teachers have a responsibility to provide enrichment opportunities to students, and a responsibility to report to parents in a way that acknowledges the students' accomplishments.

This statement suggests some development band activities — students and teachers will identify many more fields worth pursuing. A valid development band activity is a significant new piece of work, not merely an extra "extension example" or set of examples. Teachers of senior secondary students will need to establish clear criteria for the evaluation of development band work and a system which allows some basis for comparability between the work of different students. Students might undertake "units" of development band work, for example.

Development band activities should include a measure of self-assessment. Students should be encouraged to set their own goals in this work and to be self-critical. They should keep a portfolio of their development activities, including the goals they had set, their assessments, and their teachers' assessments, as a record of their extended progress. Possession of the portfolio should, among other things, ensure that students do not repeat development work in later years.

Courses for Senior Secondary School

The motivations of students studying mathematics in senior secondary school range from the necessity to consolidate more basic mathematical skills, through the requirement to establish vocationally oriented mathematical skills, to the intention to go on to tertiary studies involving some form of mathematics. It is expected that schools will construct courses according to the particular needs of these diverse groups of students.

The School Certificate examination will be based on objectives up to level 6.

The New Zealand Qualifications Authority is expected to credit units of learning for National Certificate Qualification based on the level 6, level 7, and level 8 objectives in this statement. Units based on objectives at level 6 and lower will cater for senior school students for whom an academic course of study based on levels 7 and 8 is not appropriate.

The Universities Bursary examinations will be based on appropriate subsets of the objectives up to level 8.*

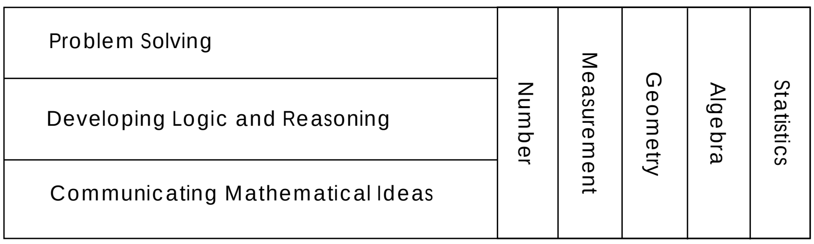
It is not expected that all sixth and seventh form students will do all of the mathematics suggested for levels 7 and 8 in this statement.

* The prescriptions for the Universities Bursary examinations are published by the New Zealand Qualifications Authority.

Mathematical Processes

The mathematics curriculum intended by this statement will provide opportunities for students to:

- develop flexibility and creativity in applying mathematical ideas and techniques to unfamiliar problems arising in everyday life, and develop the ability to reflect critically on the methods they have chosen;
- become effective participants in problem-solving teams, learning to express ideas, and to listen and respond to the ideas of others;
- develop the skills of presentation and critical appraisal of a mathematical argument or calculation, use mathematics to explore and conjecture, and learn from mistakes as well as successes;
- develop the characteristics of logical and systematic thinking, and apply these in mathematical and other contexts, including other subjects of the curriculum;
- become confident and competent users of information technology in mathematical contexts;
- develop the skills and confidence to use their own language, and the language of mathematics, to express mathematical ideas;
- develop the knowledge and skills to interpret written presentations of mathematics .

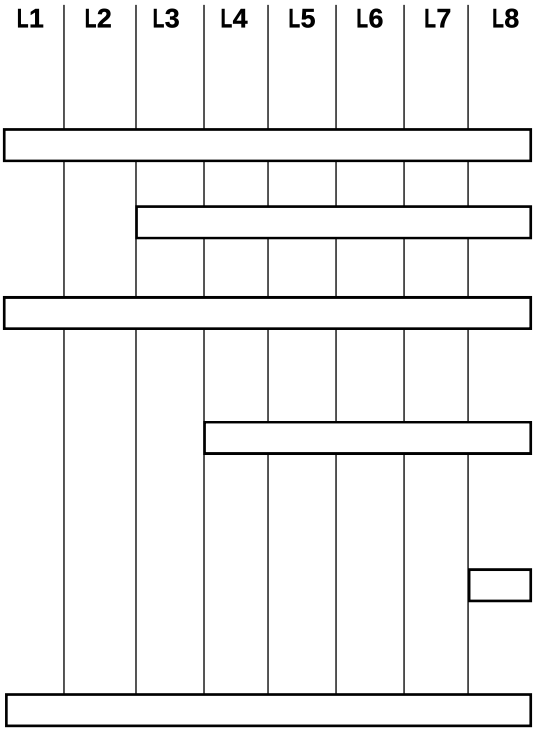


The mathematical processes skills — problem solving, reasoning, and communicating mathematical ideas — are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.

Problem Solving

Achievement Objectives

Within a range of meaningful contexts, students should be able to:



- pose questions for mathematical exploration;
- effectively plan mathematical exploration;
- devise and use problem-solving strategies to explore situations mathematically;
- find, and use with justification, a mathematical model as a problem-solving strategy;
- devise and use with justification a mathematical model as a problem-solving strategy;
- use equipment appropriately when exploring mathematical ideas.

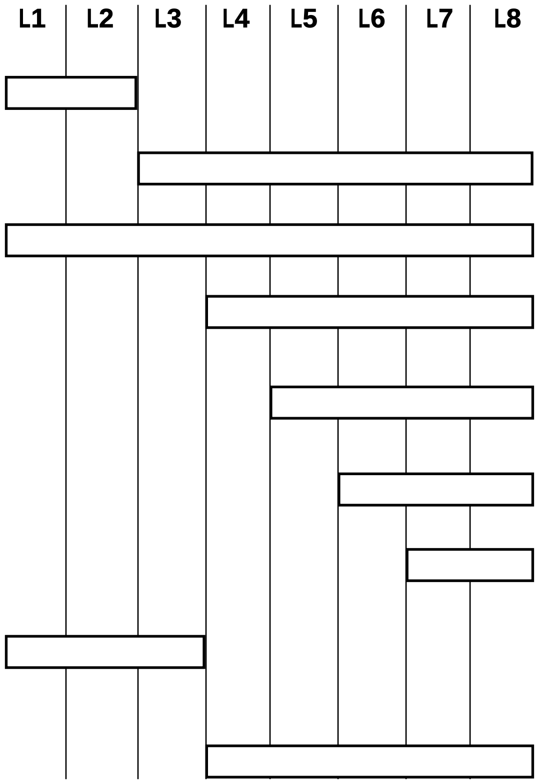
Suggested Learning Experiences

Students should be:	L1	L2	L3	L4	L5	L6	L7	L8
• posing questions and setting goals for mathematical exploration:	■	■	■	■	■	■	■	■
- exploring their own interests	■	■	■	■	■	■	■	■
- brainstorming possible questions for investigation	■	■	■	■	■	■	■	■
- predicting the outcomes of mathematical enquiry	■	■	■	■	■	■	■	■
- reflecting on their own work	■	■	■	■	■	■	■	■
- critically evaluating their work;			■	■	■	■	■	■
• developing effective planning skills by identifying component tasks, and devising, negotiating, and following sets of instructions;	■	■	■	■	■	■	■	■
• choosing and using equipment and resources (including calculators, computers, and everyday objects) appropriately;	■	■	■	■	■	■	■	■
• devising, using, and modifying problem-solving strategies:	■	■	■	■	■	■	■	■
- seeking solutions through trial and error, "acting it out", trying alternatives, and looking for patterns	■	■	■	■	■	■	■	■
- making lists and tables		■	■	■	■	■	■	■
- trying simpler cases			■	■	■	■	■	■
- being systematic, using symmetry, working backwards, making models, and using interpolation				■	■	■	■	■
- trying different approaches, eliminating possibilities, and writing equations					■	■	■	■
- using and justifying mathematical models					■	■	■	■
- examining extreme cases and using extrapolation;						■	■	■
• drawing on skills and knowledge from a range of different content areas.	■	■	■	■	■	■	■	■

Developing Logic and Reasoning

Achievement Objectives

Within a range of meaningful contexts, students should be able to:



- classify objects;
- classify objects, numbers, and ideas;
- interpret information and results in context;
- make conjectures in a mathematical context;
- generalise mathematical ideas and conjectures;
- prove or refute mathematical conjectures;
- critically follow a chain of reasoning;
- use words and symbols to describe and continue patterns;
- use words and symbols to describe and generalise patterns.

Suggested Learning Experiences

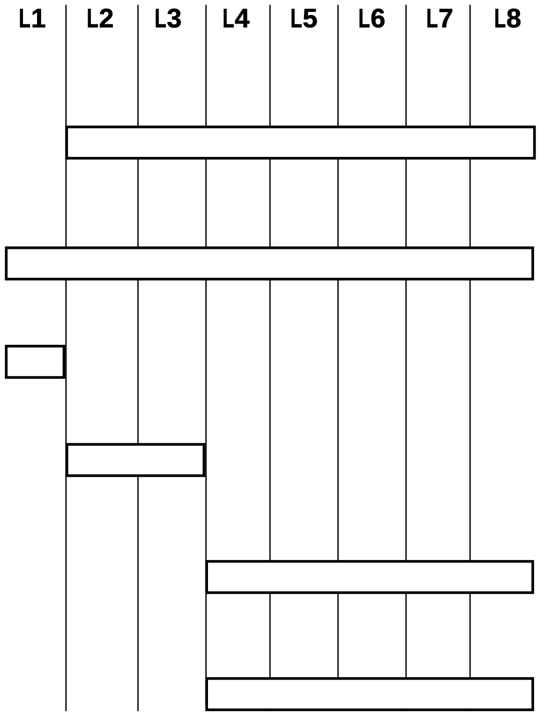
Students should be:

	L1	L2	L3	L4	L5	L6	L7	L8
• classifying and interpreting:	■	■	■	■	■	■	■	■
- categorising and sorting objects and pictures	■	■	■	■	■			
- organising and interpreting data, using diagrams, graphs, and models			■	■	■	■	■	■
- organising and interpreting tables			■	■	■	■	■	■
- interpreting symbols;				■	■	■	■	■
• recognising and working with patterns in a variety of forms and contexts:	■	■	■	■	■	■	■	■
- describing and continuing picture patterns	■							
- describing and continuing word and number patterns		■	■	■				
- describing a rule for continuing a pattern			■	■	■	■	■	■
- generalising from patterns;				■	■	■	■	■
• developing arguments and thinking flexibly:	■	■	■	■	■	■	■	■
- making simple deductions	■	■	■	■	■	■	■	■
- making simple conjectures				■	■	■	■	■
- recognising logical arguments				■	■	■	■	■
- proving and refuting					■	■	■	■
- following a chain of reasoning					■	■	■	■
- finding logic flaws in arguments						■	■	■
- demonstrating methods of mathematical proof, including proof by contradiction and counter example							■	■
- proving by induction.								■

Communicating Mathematical Ideas

Achievement Objectives

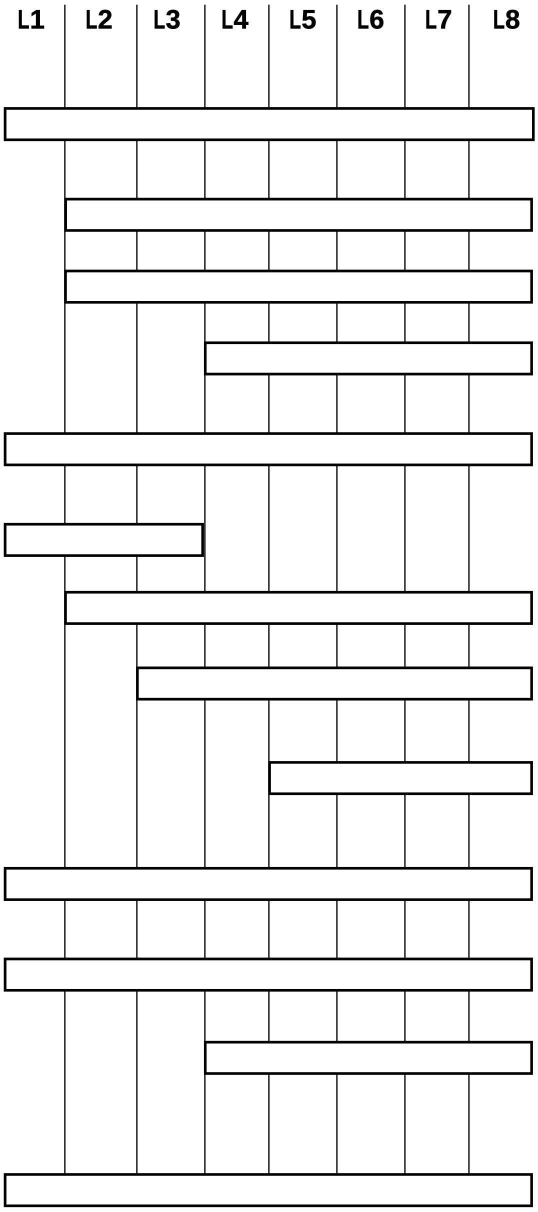
Within a range of meaningful contexts, students should be able to:



- use their own language, and mathematical language and diagrams, to explain mathematical ideas;
- devise and follow a set of instructions to carry out a mathematical activity;
- record and talk about the results of mathematical exploration;
- record, in an organised way, and talk about the results of mathematical exploration;
- record information in ways that are helpful for drawing conclusions and making generalisations;
- report the results of mathematical explorations concisely and coherently.

Suggested Learning Experiences

Students should be:

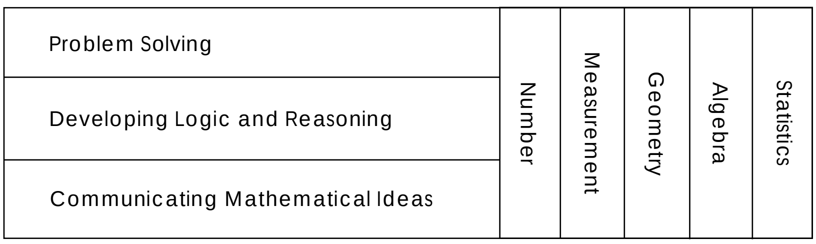


- recording in words, pictures, and concrete materials:
  - presenting diagrams (charts and graphs)
  - using symbols appropriately
  - displaying data in tables;
- presenting mathematical ideas and results to others:
  - explaining results in words and pictures
  - reporting in words and diagrams
  - making written and oral reports
  - reporting in formal mathematical language;
- explaining, discussing, and presenting arguments:
  - making clear statements
  - making logical and concise statements and deductions;
- working co-operatively as part of a group by listening attentively, generating ideas, and participating in reflective discussion.

Number

The mathematics curriculum intended by this statement will provide opportunities for students to:

- develop an understanding of numbers, the ways they are represented, and the quantities for which they stand;
- develop accuracy, efficiency, and confidence in calculating — mentally, on paper, and with a calculator;
- develop the ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements.



The mathematical processes skills — problem solving, reasoning, and communicating mathematical ideas — are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.

Number

Achievement Objectives

Exploring number

Within a range of meaningful contexts, students should be able to:

- make up, tell, and record number stories, up to 9, about given objects and sequence pictures;
- form a set of up to 20 objects;
- read and write any 2-digit whole number;
- rote count to at least 50.

Exploring computation and estimation

Within a range of meaningful contexts, students should be able to:

- make sensible estimates and check the reasonableness of answers;
- model and explain addition calculations with a sum of up to 20;
- using up to 20 objects, model and explain subtraction calculations;
- find, by practical means, one half and one quarter of a shape, and a half of a set of objects.

Suggested Learning Experiences

Exploring number

Students should be:

- developing a number sense by exploring number in the contexts of their own experiences and the world around them;
- finding examples of numbers in their everyday lives and developing numeral recognition;
- counting, ordering, and comparing numbers up to 5 (and later up to 9, and then up to 20);
- making and talking about sets up to 5 (and later up to 9, and then up to 20);
- exploring the number system from zero to 99 and beyond;
- rote counting to 99 and counting forwards and backwards to and from 99 in 1s, 2s, 5s, and 10s (both orally and using a calculator);
- exploring the idea of place value through comparing number patterns in 10s. For example: 3, 13, 23, 33, ... 93 in English, Maori,* and symbols;
- using money ($10 notes and $1 coins) and structured materials to represent numbers;
- exploring the meaning of digits in any 2-digit whole number;
- exploring the idea of a fraction;
- finding halves and quarters of everyday objects, common shapes, and sets of objects, using practical methods.

Exploring computation and estimation

Students should be:

- developing number sense by exploring estimation and computation, in the context of their own experiences and the world around them, using concrete materials, mental strategies, and calculators;
- joining 2 or more sets with a combined total of up to 9, and later 20;
- separating a set of up to 9 objects (later up to 20 objects) into two or more parts;
- making up, telling, and recording number stories about objects and sequence pictures;
- developing mental strategies for adding numbers of objects up to a total of 9 and later 20;†
- developing mental strategies for subtracting numbers of objects from up to 20;†
- making sensible estimates and checking the reasonableness of answers.

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* The Maori representations of 2-digit numbers are particularly useful for reinforcing place-value notions. For example, “rua tekau mā whā” translates to “two tens and four” (24).

† It is not suggested that students rote-learn standard algorithms for addition or subtraction at this level.

Number

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students make up and record in a variety of ways (with models, charts, displays) number stories which may arise from given sets of objects or “sequence pictures” and talk about each recording they make.

  Using this example, teachers could assess students’ ability to:
  - make up, tell, and record number stories, up to 9, about given objects and sequence pictures (N1);
  - collect everyday objects, sort them into categories, count the number of objects in each category, and display and discuss the results (S1);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP1).

- Students:
  - model 2-digit numbers, using grouped discrete objects, place-value blocks (tens and ones), and money ($10 notes and $1 coins);
  - say or read, in English and in Maori, number words for a variety of 2-digit numbers;
  - use both pencil and paper, and a calculator, to show 2-digit numbers presented orally.

  Using this example, teachers could assess students’ ability to:
  - read and write any 2-digit whole number (N1);
  - interpret information and results in context (MP1);
  - devise and use problem-solving strategies to explore situations mathematically (MP1).

- Students:
  - working in a group of four, share a lunch of a piece of fruit and a sandwich by dividing them into halves and quarters, and share packets of raisins and containers of fruit drink. They record their lunch menu, using the terms half and quarter;
  - show a half and a quarter on a geoboard in as many ways as possible, and record the results on dot paper;
  - fold paper shapes — squares, oblongs (non-square rectangles), circles, and so on — into halves and quarters. Talk about the fractions.

  Using this example, teachers could assess students’ ability to:
  - find, by practical means, one half and one quarter of a shape and half of a set of objects (N1);
  - record and talk about the results of mathematical exploration (MP1).

Sample Development Band Activities

- Students record numbers they notice in their environment within a set time, for example, a school day. Students should make up a diary record of such discoveries, including where the number was found and any information concerning the number or its associated set.
- Students create their own number stories and record them in as many different ways as they can. They comment on the advantages/disadvantages of different ways of recording.
- Students devise practical number games involving estimation to share with others. For example, one partner gives two numbers and the other states the sum or difference and then explains the method used. They check the answer, using equipment.
- Given answers, students suggest appropriate questions.

Number

Achievement Objectives

Exploring number

Within a range of meaningful contexts, students should be able to:

- read any 3-digit whole number;
- explain the meaning of the digits in 2- or 3-digit whole numbers;
- order any set of three or more whole numbers (up to 99);
- write and solve comparison problems;
- write and solve story problems which involve halves, quarters, thirds, and fifths.

Exploring computation and estimation

Within a range of meaningful contexts, students should be able to:

- make sensible estimates and check the reasonableness of answers;
- recall the basic addition and subtraction facts;
- mentally perform calculations involving addition and subtraction;
- demonstrate the ability to use the multiplication facts;
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division;
- write and solve story problems which require a choice of any combination of the four arithmetic operations.

Suggested Learning Experiences

Exploring number

Students should be:

- developing a number sense by exploring number in the context of their everyday experiences and the world around them;
- counting, recording, ordering, and comparing numbers;
- investigating number patterns with and without the aid of a calculator;
- exploring the number system from 0 to 1000 and beyond, using a calculator, structured materials, money, and measurement;
- exploring place value by using grouped discrete objects, structured materials, measurement, money, and a calculator, and by counting in Maori*;
- exploring the place of number within their own cultures;
- investigating odd and even numbers;
- investigating ways (including using a calculator) to rename numbers. For example:

$10 = 6+4$	$53 = 50+3$	$167 = 100+67$
$= 2+2+2+2+2$	$= 40+13$	$= 100+60+7$
$= 1+2+3+4$, etc.	$= 33+15+5$, etc.	$= 150+17$, etc.;

- exploring number patterns showing multiples;
- exploring fractions (halves, quarters, thirds, fifths).

Exploring computation and estimation

Students should be:

- exploring estimation and computation in the context of their everyday lives;
- working with numbers 0 to 1000 and beyond;
- performing additions and subtractions, using pencil and paper, and calculator, and making up and telling number stories about their calculations;
- investigating the special properties of 0, under addition and subtraction, and of 1, under multiplication and division;
- devising strategies to help the memorising of the addition, subtraction, multiplication, and division facts;
- developing instant recall of basic addition and subtraction facts through a programme of regular maintenance;
- devising and refining strategies for estimating and checking the reasonableness of addition, subtraction, multiplication, and division calculations;
- devising and extending mental strategies for addition, subtraction, multiplication, and division;

*The Maori representations of 2-digit numbers are particularly useful for reinforcing place-value notions. For example, "rua tekau mā whā" translates to "two tens and four" (24).

- solving comparison problems (how many more than or less than) in a story context;
- adding and subtracting money and measurements;
- exploring division as sharing;
- using a calculator and concrete materials to develop the meaning of multiplication as repeated addition, and division as repeated subtraction;
- developing recall of multiplication facts through a programme of regular maintenance;
- using a calculator to investigate answers to division problems;
- writing and solving story problems (including those involving money and measurements) which involve combinations of the 4 operations, using a calculator where necessary;
- recognising that taking a fraction of a set is a sharing (dividing) operation.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students report on an investigation of number and operations, using a calculator. For example, given three digits, say 6, 7, and 9, and one multiplication sign, use a calculator to determine which arrangement gives the largest product (for example, 6, 7, 9 gives 67 x 9, 97 x 6, etc.).

  Using this example, teachers could assess students' ability to:
  - explain the meaning of the digits in 2- or 3-digit whole numbers (N2);
  - devise and use problem-solving strategies to explore situations mathematically (MP2);
  - record and talk about the results of mathematical exploration (MP2).

- Students resolve comparison problems arising in familiar contexts. For example, they:
  - investigate the number of pets per household in the class, graph the results, and answer questions such as, "Marama has 4 pets. One of her pets had babies and Marama now has 7 pets. How many babies did her pet have?", "Peta has 7 pets. Alice has 11. How many fewer does Peta have than Alice?", "Ripeka has 9 pets. This is 6 fewer than Olivia's number of pets. How many pets has Olivia?" "Toni has 13 pets. She has 5 more than Jenny. How many pets does Jenny have?";
  - compare numbers of people in families using the words "more than", "fewer than", and "difference", and record the findings on posters.

  Using this example, teachers could assess students' ability to:
  - write and solve comparison problems (N2);
  - devise and use problem-solving strategies to explore situations mathematically (MP2);

- record and talk about the results of mathematical exploration (MP2).

- Students use a calculator to solve story problems such as the following:
  - Huia is 8 years old today. She knows that there are 365 days in a year. How many days old is Huia? How many weeks old (hours? minutes?) is Huia?
  - Make up multiplication stories with an answer (product) of, for example, 24.
  - Write division stories with an answer of, for example, 2.

  Using this example, teachers could assess students' ability to:
  - write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (N2);
  - make sensible estimates and check the reasonableness of answers (N2);
  - devise and use problem-solving strategies to explore situations mathematically (MP2);
  - record and talk about the results of mathematical exploration (MP2).

- Students estimate and calculate in unfamiliar contexts. For example, if A is worth 1, B is worth 2, etc., find the word with fewer than 5 letters where the sum of the letters has the greatest value.

  Using this example, teachers could assess students' ability to:
  - devise and use problem-solving strategies to explore situations mathematically (MP2);
  - interpret information and results in context (MP2).

- Students solve simple fraction problems such as "A third of a class of 27 go to a culture group practice. How many is this?"

  Using this example, teachers could assess students' ability to:
  - write and solve story problems which involve halves, quarters, thirds, and fifths (N2);
  - devise and use problem-solving strategies to explore situations mathematically (MP2);
  - record and talk about the results of mathematical exploration (MP2).

Sample Development Band Activities

- Students invent, and make up rules for, games based on a set of numeral cards or a calculator.
- Students document and explain how the use of Maori language supports the idea of place value.
- Students explore numbers in other bases, say base 2.

Number

Achievement Objectives

Exploring number

Within a range of meaningful contexts, students should be able to:

- explain the meaning of the digits in any whole number;
- explain the meaning of the digits in decimal numbers with up to 3 decimal places;
- order decimals with up to 3 decimal places.

Exploring computation and estimation

Within a range of meaningful contexts, students should be able to:

- make sensible estimates and check the reasonableness of answers;
- recall the basic multiplication facts;
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations;
- solve practical problems which require finding fractions of whole number and decimal amounts.

Suggested Learning Experiences

Exploring number

Students should be:

- developing a number sense by exploring number in the context of their everyday experiences and the world around them, and using numbers to explore events in their own lives;
- extending their understanding of the number system;
- using calculators to develop an understanding of whole numbers beyond 1000;
- exploring the use of fractions and decimals in society;
- developing meaning for decimal place values, using concrete models and a calculator, for example, using place-value blocks, repeated division by 2 leading to 4, 2, 1, 0.5, 0.25, 0.125, 0.0625, ...
- saying decimals, for example, 1.25 is one and twenty-five hundredths, or one and two tenths and five hundredths, or one hundred and twenty-five hundredths; 1.25 is read "one point two five", not "one point twenty-five";
- writing decimals in words and symbols;
- using calculators and number lines (rulers and other linear scales) to compare and order decimals;
- exploring number patterns which involve both whole numbers and decimals;
- investigating possible ways of renaming numbers using decimals. For example,

$$\begin{aligned}1 &= 0.4 + 0.6\\ &= 0.2 + 0.2 + 0.2 + 0.2 + 0.2\\ &= 0.12 + 0.88\\ &= 0.346 + 0.237 + 0.417\\ &= \ldots\end{aligned}$$

Exploring computation and estimation

Students should be:

- developing a number and computation sense by exploring estimation and computation in the context of their everyday lives;
- maintaining addition and subtraction facts;
- demonstrating the instant recall of basic multiplication facts;
- working with whole numbers and decimals;
- using a calculator and mental methods to add or subtract numbers;
- making sensible estimates and checking the reasonableness of answers;
- writing and solving story problems involving one or more of the four arithmetic operations;
- using calculators, concrete materials, and mental methods to find fractions of whole number and decimal amounts (including money and measurements).

Number

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students:
  - explain in their own words which four in 0.444 has the greatest value;
  - working in groups, measure their heights in metres (to 2 decimal places) and order the group from tallest to shortest;
  - solve problems from information presented in table form. For example, the scores from a gymnastics competition are:

Pukerua Bay Gym Competition				
	Floor	Horizontal Bar	Parallel Bars	Vault
Tony	8.20	7.25	8.30	9.55
Jim	7.55	8.50	7.30	4.50
John	6.80	9.15	6.25	7.25
Warren	6.20	6.20	5.65	8.30
Dave	8.20	5.35	8.50	6.25
Murray	6.00	7.50	5.50	6.50

Put the scores for the vault in order. Put John's scores in order. Find the total score for each gymnast. Order the total scores from highest to lowest. Find the difference between the highest and the lowest scores. If Warren had scored 9.25 on each activity, what would his total score have been? Find the difference between John's total score and Warren's total score and work out by how much Warren has to improve on each activity to equal John's score.

Using this example, teachers could assess students' ability to:
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (N3);
- order decimals with up to 3 decimal places (N3);
- devise and use problem-solving strategies to explore situations mathematically (MP3).

- Students use a calculator to explore different ways of representing a number as the sum of decimal numbers (1 is a good example).

Using this example, teachers could assess students' ability to:
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (N3);
- devise and use problem-solving strategies to explore situations mathematically (MP3);
- use equipment appropriately when exploring mathematical ideas (MP3).

- Students calculate fractions of decimal amounts. for example, six children decide to save $\frac{2}{5}$ of their pocket money each week. How much will each save?

Pocket Money List	
Nani	$2.00
Hana	$2.50
Ben	$2.60
Rhonda	$3.00
Diane	$2.80
Sarah	$3.20

Using this example, teachers could assess students' ability to:
- solve practical problems which require finding fractions of whole number and decimal amounts (N3);
- devise and use problem-solving strategies to explore situations mathematically (MP3);
- use equipment appropriately when exploring mathematical ideas (MP3).

Sample Development Band Activities

- Students compare the appeal and mathematical content of a number of "mathematical" games.
- Having solved number problems, such as those below, students invent other number problems based on the same theme and draw generalisations.
  - How many ways can you arrive at a sum of 10, using two dice?
  - Which combination of numbers that add up to 12 produces the greatest product?
- Students investigate numbers associated with the planetary system, for example, how many planets there are, their masses, and their greatest distances from the sun.
- Students find divisibility tests for all numbers under 12 (except 7).
- Students explore arithmetic in, say, base 3.

Number

Achievement Objectives

Exploring number

Within a range of meaningful contexts, students should be able to:

- explain the meaning of negative numbers;
- explain the meaning and evaluate powers of whole numbers;
- find fractions equivalent to one given;
- express a fraction as a decimal, and vice versa;
- express a decimal as a percentage, and vice versa;
- express quantities as fractions or percentages of a whole.

Exploring computation and estimation

Within a range of meaningful contexts, students should be able to:

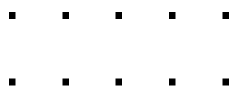
- make sensible estimates and check the reasonableness of answers;
- write and solve problems involving decimal multiplication and division;
- find a given fraction or percentage of a quantity;
- explain satisfactory algorithms for addition, subtraction, and multiplication;
- demonstrate knowledge of the conventions for order of operations.

Suggested Learning Experiences

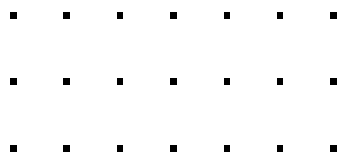
Exploring number

Students should be:

- developing their number sense by exploring number in the context of the world around them;
- using number to explore events in their own lives and cultures;
- extending their understanding of the number system;
- investigating the use of the calculator memory key;
- exploring factors of numbers by investigating rectangular (composite) numbers and line (prime) numbers;



10 = 2 x 5



21 = 3 x 7

5 is prime

- exploring equivalent fractions;
- exploring a variety of number patterns, including powers and roots;
- relating fractions to decimals;
- using calculators to investigate patterns in recurring decimals;
- talking about the use of percentages in everyday contexts;
- using calculators to develop meaning for percentages;
- saying decimals as percentages, for example, 0.43 is 43%;
- using calculators to help explore meanings for positive and negative numbers;
- inventing and investigating contexts for negative numbers;
- investigating the results of multiplying and dividing decimals by powers of 10 (10, 100, 1000 ...).

Exploring computation and estimation

Students should be:

- developing their number and computation sense by exploring estimation and computation in the context of their everyday lives;
- maintaining basic facts;
- using money and measurements to explore sensible rounding techniques;
- exploring the outcomes of multiplication and division, using decimals;
- devising and using strategies for estimating the results of computations involving decimals;
- solving and writing problems involving the multiplication and division of decimals;
- finding percentages of quantities;
- exploring and establishing conventions for order of operations.

Number

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students estimate proportions in surveys before making detailed analyses. For example, a survey of interests among the class indicates that television watching is the most common. They survey favourite television programmes, and then estimate proportions, expressing them as fractions, decimals, and percentages.

  Using this example, teachers could assess students' ability to:
  - make sensible estimates and check the reasonableness of answers (N4);
  - express a decimal as a percentage, and vice versa (N4);
  - find a given fraction or percentage of a quantity (N4);
  - find fractions equivalent to one given (N4);
  - express a fraction as a decimal, and vice versa (N4);
  - make statements about implications and possible actions consistent with the results of a statistical investigation (S4);
  - estimate the relative frequencies of events and mark them on a scale (S4).

- Students translate measurements taken from everyday activities into proportions. For example, students might use a calculator and a road map of New Zealand to find decimal proportions of the total distance covered at various stages on a journey. For example, on a journey from Hamilton to Wellington, what proportion of the trip has been covered by the time the traveller reaches Bulls?

  Using this example, teachers could assess students' ability to:
  - express quantities as fractions or percentages of a whole (N4);
  - read ... a variety of ... timetables and charts (M4).

- Students make a presentation to the class involving an explanation of negative numbers. For example, a group investigates and reports to the class on the daily temperatures at the Hermitage (near Mt Cook) and clearly explains the meaning and relationship of the negative numbers involved.

  Using this example, teachers could assess students' ability to:
  - explain the meaning of negative numbers (N4);
  - interpret information and results in context (MP4).

- Students perform calculations in practical contexts. For example, a runner can run 100 metres in 10.7 seconds. How long would the runner take to cover 1500 metres at the same rate? Compare the answer with realistic times for the 1500 metres.

  Using this example, teachers could assess students' ability to:
  - write and solve problems involving decimal multiplication and division (N4);
  - use their own language, mathematical language, and diagrams to explain mathematical ideas (MP4);
  - perform calculations with time ... (M4).

- Students demonstrate understanding of the correct order of arithmetic operations by, for example, inserting brackets where necessary to make true statements, such as in 7 + 2 x 3 = 27, 7 + 2 x 3 = 13.

  Using this example, teachers could assess students' ability to:
  - demonstrate knowledge of the conventions for order of operations (N4).

- Students work in pairs to categorise the numbers 1 to 40 by their factors, using the notion of rectangular numbers.

  Using this example, teachers could assess students' ability to:
  - write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (N3);
  - use words and symbols to describe and generalise patterns (MP4);
  - make conjectures in a mathematical context (MP4).

- Students explore, analyse, and describe a numerical investigation. For example, a container of rods contains only 5 cm and 7 cm lengths. If they are placed end to end, what lengths cannot be made?

  Using this example, teachers could assess students' ability to:
  - write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (N3);
  - devise and use problem-solving strategies to explore situations mathematically (MP4).

Sample Development Band Activities

- Students solve magic squares. They then create their own, for example, all 3 x 3 magic squares with the sum of 15. They investigate how magic squares are constructed and explore the history of them. In addition, they could explore magic cubes.
- Students investigate and report on properties of "special" numbers. For example:
  - they make and test conjectures arising from observing the sequence 4 = 3+1, 6 = 3+3, 8 = 5+3, 10 = 5+5, ...;
  - they explore the properties of palindromic numbers (373, 2882, and 62126 are palindromes);
  - they explore "twin primes", for example, they make and test conjectures about the sequence of even numbers generated by an ordered sequence of twin primes. (5 6 7, 11 12 13, 17 18 19, ...).
- Students investigate and report on modulo (clock) arithmetic.
- Students explore cryptography.

Number

Achievement Objectives

Exploring number

Within a range of meaningful contexts, students should be able to:

- convert numbers expressed in standard form to ordinary form, and vice versa;
- round numbers sensibly;
- express the values of square roots in approximate and exact forms.

Exploring computation and estimation

Within a range of meaningful contexts, students should be able to:

- make sensible estimates and check the reasonableness of results;
- solve practical problems involving decimals and percentages;
- solve problems involving positive and negative numbers, using practical activities or models if needed;
- express one quantity as a percentage of another;
- increase and decrease quantities by given percentages, including mark up, discount, and GST;
- share quantities in given ratios.

Suggested Learning Experiences

Exploring number

Students should be:

- developing their number sense by exploring number in the context of the world around them;
- expressing numbers in index form;
- exploring the use of standard form in practical contexts;
- converting numbers from ordinary form to standard form, and vice versa;
- exploring the use of ratio in everyday contexts;
- developing meaning for ratio by comparing two like quantities;
- investigating equivalent ratios;
- recording ratios as $\frac{a}{b}$ and a:b.

Exploring computation and estimation

Students should be:

- developing their number and computation sense by exploring estimation and computation in the context of their everyday lives;
- making sensible estimates and checking the reasonableness of answers to problems set in practical contexts;
- showing that they are aware when the answer shown by a calculator is not sensible;
- exploring and using rounding techniques, including significant figure and decimal place rounding;
- developing mental strategies for adding, subtracting, multiplying, and dividing positive and negative numbers, using a calculator, a variety of models, and other approaches;
- solving problems involving positive and negative numbers;
- finding percentages of quantities, and one quantity as a percentage of another, using a calculator where appropriate;
- increasing and decreasing quantities by given percentages;
- investigating profit and loss related to cost price and selling price;
- sharing quantities in given ratios;
- writing and solving their own computational problems;
- developing and using strategies for adding and subtracting fractions.

Number

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students convert correctly to and from standard form, using examples taken from other studies. For example:
  - The speed of light is approximately 299 800 000 m/s. Express this in standard form.
  - The mass of an electron is approximately 9.11×10^{-31} kg. Write this as an ordinary number.

  Using this example, teachers could assess students' ability to:
  - convert numbers expressed in standard form to ordinary form, and vice versa (N5).
- Students round numbers which arise from practical situations. For example, 7 people have a restaurant meal. They decide to share the bill of $187.45 equally. How much does each person pay? Students explain their rounding.

  Using this example, teachers could assess students' ability to:
  - solve practical problems involving decimals and percentages (N5);
  - round numbers sensibly (N5).
- Students use positive and negative numbers to write questions that result in negative answers, for example, $^-4$ could be written as $^-1 + {}^-3$, $^-7 - {}^-3$, $^-2 \times 2$, etc.

  Using this example, teachers could assess students' ability to:
  - solve problems involving positive and negative numbers, using practical activities or models if needed (N5);
  - pose questions for mathematical exploration (MP5).
- Students carry out calculations involving percentages, ratios, and proportions derived from everyday situations. For example:
  - Pene wants to buy a motor bike. She needs to borrow $1200. The bank charges 9.5% interest per year. How much interest does she pay if the loan is repaid in a lump at the end of the year? Students report on an investigation of other ways of calculating interest and repayment options.
  - While on holiday in Hawaii, Reina bought a surfboard for US $456. Use exchange rates from a newspaper or teletext to calculate how much she will have to pay in NZ dollars.
  - Two families buy 15 cubic metres of firewood and split it in the ratio 3:2. How much will each family receive?

  Using this example, teachers could assess students' ability to:
  - solve practical problems involving decimals and percentages (N5);
  - increase and decrease quantities by given percentages, including mark up, discount, and GST (N5);
  - share quantities in given ratios (N5);
  - interpret information and results in context (MP5);
  - pose questions for mathematical exploration (MP5).

- Students generalise their findings from number problems. For example, devise a quick method for adding a set of consecutive whole numbers. Find a method which works and explain why.

  Using this example, teachers could assess students' ability to:
  - solve problems involving positive (and negative) numbers, using practical activities or models if needed (N5);
  - devise and use problem-solving strategies to explore situations mathematically (MP5);
  - make conjectures in a mathematical context (MP5);
  - use their own language, mathematical language, and diagrams to explain mathematical ideas (MP5).

Sample Development Band Activities

- Students make up flow charts for numerical applications, such as an algorithm for calculating tax payable. Students could write a computer program or use a spreadsheet to generate tax tables.
- Students investigate number problems. For example, given access to 3 cent and 7 cent stamps only, what amounts of postage could be made up exactly? What about a cent and b cent stamps?
- Students investigate the representation of any integer as a product of primes, and show how to use this property in determining greatest common divisors, and lowest common multiples.
- 1089 x 9 = 9801. Explore.

Number

Achievement Objectives

Exploring computation and estimation

Within a range of meaningful contexts, students should be able to:

- perform basic operations on fractions and mixed numbers;
- classify numbers as whole, integer, rational, or irrational;
- devise a strategy to solve a number problem;
- estimate and calculate answers, making efficient use of a calculator, where appropriate, as part of solving a problem;
- discuss the reasonableness and meaning of the answers obtained in solving a problem.

Suggested Learning Experiences

Exploring computation and estimation

Students should be:

- continuing the development of their number knowledge, estimation, and computation skills while investigating and solving everyday problems;
- investigating surds and other irrational numbers;
- using a calculator efficiently;
- reaching sensible decisions in making approximations and roundings;
- judging reasonableness of answers obtained;
- interpreting the meaning of the answers obtained;
- developing and using strategies for multiplication and division of fractions.

Number

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students work through problems derived from everyday situations which involve a range of number skills. For example:

  - They adapt a recipe for 3 people to provide enough for 8 people.
  - They plan a discotheque to raise $100, estimating costs and a reasonable ticket price. Then they make graphs of costs ($) vs number of people, and on the same axes graph income ($) vs number of people. They use the graph to determine how many tickets need to be sold (a) to break even (b) to make $100.
  - Students investigate the calculation of savings interest.

  Using this example, teachers could assess students' ability to:
  - perform basic operations on fractions and mixed numbers (N6);
  - devise a strategy to solve a number problem (N6);
  - estimate and calculate answers, making efficient use of a calculator, where appropriate, as part of solving a problem (N6);
  - discuss the reasonableness and meaning of the answers obtained in solving a problem (N6).

- Students solve optimisation problems. For example, as part of a fundraising effort, the class is to make 200 pizzas for sale. The bases cost $90, cheese costs $62, sauce and toppings cost $68 altogether. It costs 5.4c per five pizzas to cook them. At $1.50 a pizza, all the pizzas will sell, but for every 50c increase in selling price there will be a 20% reduction in sales. The students calculate the best selling price for maximum profit.

  Using this example, teachers could assess students' ability to:
  - devise a strategy to solve a number problem (N6);
  - estimate and calculate answers, making efficient use of a calculator, where appropriate, as part of solving a problem (N6);
  - pose questions for mathematical exploration (MP6);
  - devise and use problem-solving strategies to explore situations mathematically (MP6).

- Students report an investigation of Pythagorean triples: (i) 3, 4, 5 (ii) 5, 12, 13 (iii) 7, 24, 25.

  Using this example, teachers could assess students' ability to:
  - devise a strategy to solve a number problem (N6);
  - estimate and calculate answers, making efficient use of a calculator, where appropriate, as part of solving a problem (N6);
  - make conjectures in a mathematical context (MP6);
  - prove or refute mathematical conjectures (MP6);

- report the results of mathematical explorations concisely and coherently (MP6).

- Students use fraction manipulation in context. For example, a punch for a party has 4 litres of ginger ale, 2 litres of orange juice, and $\frac{1}{2}$ litre of cold tea. How much extra orange juice would need to be added to bring it to 40% of the total mixture?

  Using this example, teachers could assess students' ability to:
  - perform basic operations on fractions and mixed numbers (N6);
  - devise a strategy to solve a number problem (N6).

Sample Development Band Activities

- Students investigate the calculation of interest in mortgage and loan repayments, and investment dividends.
- Students devise algorithms for calculating values of surds to a large number of decimal places. They should implement their algorithm, using a calculator or computer.
- Students design and implement an efficient algorithm for sorting a list of random numbers into order.

Measurement

The mathematics curriculum intended by this statement will provide opportunities for students to:

- develop knowledge and understanding of systems of measurement and their use and interpretation;
- develop confidence and competence in using instruments and measuring devices;
- predict and calculate the effects of changes in variables and rate of change of variables on systems representable by simple mathematical models.

	Number	Measurement	Geometry	Algebra	Statistics
Problem Solving					
Developing Logic and Reasoning					
Communicating Mathematical Ideas					

The mathematical processes skills — problem solving, reasoning, and communicating mathematical ideas — are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.

Measurement

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- order and compare lengths, masses, and volumes (capacities), and describe the comparisons, using measuring language;
- measure by counting non-standard units;
- compare the values of coins and notes;
- read prices.

Developing concepts of time, rate, and change

Within a range of meaningful contexts, students should be able to:

- read aspects of time, including days of the week and clocks (to hours and half hours).

Suggested Learning Experiences

Estimating and measuring

Students should be:

- exploring how measurements are made in a variety of cultural situations, and experiencing situations where values are put on measured quantities. For example, 250 ml of sauce, $2.75 for a carton of milk, 2 metres of curtain material;
- investigating length, mass, volume, temperature, and money, and making comparisons, using appropriate language, for example, longer, hotter, heavier, more;
- estimating and measuring with non-standard units to find length, area, mass, and volume;
- using a variety of measures in everyday situations, for example, in following simple recipes.

Developing concepts of time, rate, and change

Students should be:

- exploring time, for example, using a calendar, and using the language of time, growth, change, and speed such as, faster, slower, before, after, bigger, smaller, newer, older.

Measurement

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students arrange in order, from lightest to heaviest, a set of similar containers filled with different objects: buttons, blocks, shells, corks, bottle tops, and so on. Estimate first and then check, using a balance. Write and talk about the results, and draw pictures to illustrate the ordering.

  Using this example, teachers could assess students' ability to:
  - order and compare lengths, masses, and volumes (capacities) and describe the comparisons, using measuring language (M1);
  - record and talk about the results of mathematical exploration (MP1).

- Students measure short lengths from a baseline, using different children's shoes going heel to toe. Talk about why the end points are different and make statements about who has the longest shoes; the shortest shoes; the middle-sized shoes.

  Using this example, teachers could assess students' ability to:
  - order and compare lengths, masses, and volumes (capacities) and describe the comparisons, using measuring language (M1);
  - measure by counting non-standard units (M1);
  - interpret information and results in context (MP1);
  - record and talk about the results of mathematical exploration (MP1).

- Students make a chart to illustrate the weather on each day of a week. Talk about the weather during the week, focusing on the sequence of days. For example, "What was the weather like on Tuesday ?", "What was it like on the day after Wednesday?"

  Using this example, teachers could assess students' ability to:
  - read aspects of time, including days of the week and clocks (to hours and half hours) (M1);
  - record and talk about the results of mathematical exploration (MP1).

Sample Development Band Activities

- Students explore methods for comparing the capacities of containers of different shapes. They discuss the appropriateness of their methods.
- Students plant fast-growing seeds. They measure and describe their growth over time, and record the information in their own way.
- Children classify and record classroom items in terms of the devices they would choose to measure them. They explain their classification.
- Children track a situation every quarter of an hour during the day, and keep a record of their results.

Measurement

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- carry out practical measuring tasks, using appropriate metric units for length, mass, and capacity;
- give change for sums of money;
- represent a sum of money by two or more different combinations of notes and coins.

Developing concepts of time, rate, and change

Within a range of meaningful contexts, students should be able to:

- read time and know the units of time — minute, hour, day, week, month, and year.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- learning about a variety of measuring devices and units by talking with others, for example, grandparents;
- investigating the uses of measurement;
- exploring the representation of qualitative data, such as feelings and opinions, on a simple scale;
- comparing lengths, masses, volumes (capacities), and temperature, using phrases such as "is less than", "is greater than", and "is equal to", and using standard and non-standard units;
- solving measurement problems involving length, mass, and volume (capacity), in context, by estimating measurements, and counting with non-standard units and metric units (metre, centimetre, kilogram, gram, litre, and millilitre);
- solving measurement problems involving area and volume, in context, by estimating and counting with non-standard units (regular and irregular shapes);
- developing an understanding of the size of a metre, kilogram, and litre by working with these measurements in a variety of contexts;
- reading scales, for example, kitchen scales, or a parking meter;
- solving problems in context, and naming upper and lower values for estimates;
- investigating shopping problems and prices, including giving change in shopping.

Developing concepts of time, rate, and change

Students should be:

- making and using unconventional instruments to measure time, such as a water clock, a candle-timer, a sand-timer, and a sundial;
- estimating time intervals and working with units of time (minute, hour, day, week, month, year), including telling time on digital and analogue clocks;
- exploring ideas of growth, speed, and change.

Measurement

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students working in pairs measure body parts. For example, they measure arm length, distance around wrist, distance around head, and present their measurements on a graph (for example, a "body garden" graph where lengths are represented by the vertical stems of flowers). Talk about the graph to others.

  Using this example, teachers could assess students' ability to:
  - carry out practical measuring tasks, using appropriate metric units for length, mass, and capacity (M2);
  - collect and display category data and whole number data in pictograms, tally charts, and bar charts, as appropriate (S2);
  - talk about the features of their own data displays (S2);
  - interpret information and results in context (MP2);
  - use their own language, mathematical language, and diagrams to explain mathematical ideas (MP2).

- Students, working in groups, make a time chart showing the sequence of important events that happen during their day.

  Using this example, teachers could assess students' ability to:
  - read time and know the units of time — minute, hour, day, week, month, and year (M2);
  - record in an organised way and talk about the results of mathematical exploration (MP2).

Sample Development Band Activities

- Children select and time the duration of various tasks, and record, compare, and comment on the results.
- Children seek out as many time devices as they can within, for example, a 24-hour period. They record the type of device and the time shown at discovery. They present and discuss their findings.
- Children create their own calendar and record differences between theirs and the standard calendar.
- Students explore the measurement units used in recipes, for example, abbreviations, comparisons of specific and non-specific measurement (pinch of salt, knob of butter, tsp baking powder, 30 g sugar), and translations from one measure to another. They record and share their findings.
- Students explore the "drinking problem". What amounts of liquid can be accurately measured out, using only a 3-litre and a 5-litre jug?

Measurement

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates;
- perform measuring tasks, using a range of units and scales.

Developing concepts of time, rate, and change

Within a range of meaningful contexts, students should be able to:

- read and interpret everyday statements involving time;
- show analogue time as digital time, and vice versa.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- comparing the uses of measurement in different families, cultures, societies, and eras;
- exploring timetables and charts, and discussing the use of electronic measuring devices, for example, supermarket scales;
- measuring qualitative data, such as feelings and opinions, using a simple scale;
- developing an understanding of measurement and solving measurement problems;
- estimating measurements and using units of length, mass, and volume (including mm, tonne, ml), and checking by reading scales;
- developing an understanding of length, mass, and volume for everyday situations, for example, the height and mass of a nine-year-old person is about 120 cm and 40 kg respectively;
- exploring, estimating, and checking units for area (square cm, square m) and temperature (° C);
- using calculations in practical tasks, including mixing and interchanging units;
- describing the result of a measurement task as coming between a lower and an upper value.

Developing concepts of time, rate, and change

Students should be:

- using analogue and digital clocks to measure time, converting between analogue and digital expressions of time, and using seconds in practical contexts, for example, fitness circuits;
- exploring and comparing change, for example, growth, temperature, heartbeats.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students, working in pairs, prepare a set of instructions for an orienteering trail indicating distance and direction and designed for participants to reach a specific destination in the school grounds.

  Using this example, teachers could assess students' ability to:
  - demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (M3);
  - perform measuring tasks, using a range of units and scales (M3);
  - devise and follow a set of instructions to carry out mathematical activity (MP3).

- Students investigate how much boys' shoes are bigger than girls' shoes, using ideas of length, width, perimeter (distance around), area, and capacity.

  Using this example, teachers could assess students' ability to:
  - demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (M3);
  - perform measuring tasks, using a range of units and scales (M3);
  - devise and use problem-solving strategies to explore situations mathematically (MP3);
  - record in an organised way and talk about the results of mathematical exploration (MP3).

- Students investigate the statement that tall people have a higher pulse rate (number of heartbeats per minute) than short people.

  Using this example, teachers could assess students' ability to:
  - perform measuring tasks, using a range of units and scales (M3);
  - read and interpret everyday statements involving time (M3);
  - write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (N2);
  - devise and use problem-solving strategies to explore situations mathematically (MP3);
  - interpret information and results in context (MP3);
  - record in an organised way, and talk about, the results of mathematical exploration (MP3).

Sample Development Band Activities

- Students create a measuring system or device to measure, for example, the thickness of a hair, the distance a pen travels before it runs out, the distance a student walks in a day. They present their system and discuss its effectiveness.
- Students create a "favourite recipes" book by selecting, testing, and recording recipes which appeal to them. They devise a classification system for their recipes, including a "healthy food" category, and make a cross-referenced table of contents.
- Students investigate capacities in unusual situations, for example, how many 1-metre cubes would fit into the classroom?

Measurement

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- carry out measuring tasks involving reading scales to the nearest gradation;
- calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length;
- read and construct a variety of scales, timetables, and charts;
- design and use a simple scale to measure qualitative data.

Developing concepts of time, rate, and change

Within a range of meaningful contexts, students should be able to:

- perform calculations with time, including 24-hour clock times.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- exploring the uses of measurement in work, sport, at home, and in leisure, and discussing the need for measurement in our world, for example, the measurement of global warming and pollution, and measurement in dressmaking and in construction;
- developing an understanding of measurement and solving measurement problems in context;
- exploring the approximate nature of measurement, and the need for rounding to give sensible results;
- estimating and selecting appropriate units for measurement tasks involving length, mass, volume, time, area, and temperature;
- exploring the relationship between perimeter and area for figures such as rectangles, and exploring the relationship between the circumference and the diameter of a circle;
- consolidating an understanding of the concepts of area and volume, and using hectare and cubic metre;
- investigating the areas of squares, leading to squaring and finding square roots;
- understanding equivalence of units, for example, that 1 cubic decimetre of water, 1 litre of water, and 1 kg of water are equivalent quantities;
- recording measurements and making conversions between units within the metric system, using decimals and calculators where appropriate;
- constructing and using timetables and charts, and exploring the use of measuring devices which use newer technologies (lasers, decibel meters, and so on);
- interpreting diagrams involving the reading of scales, maps, plans, and so on;
- measuring qualitative data, such as attitudes, feelings, opinions, and behaviours, on simple scales, and discussing the limitations of this form of measurement.

Developing concepts of time, rate, and change

Students should be:

- reading and interpreting a 24-hour clock, and discussing the differences between 12-hour and 24-hour time and the reasons for using 24-hour time;
- exploring concepts of change, including the use of units, for example, pulse rate, birth rate, power consumption, phone charges, speed.

Measurement

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students:
  - time and cost a toll call by using toll call price charts and maps, or
  - construct a fishing calendar, using resources such as a Maori lunar calendar, or tide charts, or
  - plan an overseas trip which involves changing aircraft and time zones, using airline timetables.

  Using this example, teachers could assess students' ability to:
  - read and construct a variety of scales, timetables, and charts (M4);
  - perform calculations with time, including 24-hour clock times (M4);
  - write and solve problems involving decimal multiplication and division (N4);
  - interpret information and results in context (MP4).

- Students investigate the effect of changing the mass or length of a pendulum on its period (time for a complete back and forth swing).

  Using this example, teachers could assess students' ability to:
  - carry out measuring tasks involving reading scales to the nearest gradation (M4);
  - record information in ways that are helpful for drawing conclusions and making generalisations (MP4);
  - report the results of mathematical explorations concisely and coherently (MP4).

- Students design and construct, using card, a 1-litre container and a $\frac{1}{2}$-litre container.

  Using this example, teachers could assess students' ability to:
  - carry out measuring tasks involving reading scales to the nearest gradation (M4);
  - calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (M4);
  - design the net and make a simple polyhedron to specified dimensions (G4);
  - write and solve problems involving whole numbers and decimals, and requiring a choice of one or more of the four arithmetic operations (N3);
  - devise and use problem-solving strategies to explore situations mathematically (MP4);
  - use equipment appropriately when exploring mathematical ideas (MP4).

Sample Development Band Activities

- Students develop methods for estimating with reasonable accuracy the quantities used in everyday situations, for example, the number of bricks used in a building.
- Students investigate the history and the uses of π. They investigate relationships in which π is involved and algorithms which have been used to establish approximate values for π.
- Students investigate and report on systems for measuring astronomical distances.
- Students invent and construct a novel device to measure time. They present a report on the effectiveness and usefulness of the device.
- Students produce a quantitative report on a practical conservation theme, for example, they might research the amount of water commonly used by households, then calculate the savings to be made by fixing a dripping tap, showering rather than bathing, or placing a brick in the cistern.

Measurement

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer;
- design and use models to solve measuring problems in practical contexts.

Developing concepts of time, rate, and change

Within a range of meaningful contexts, students should be able to:

- interpret and use information about rates presented in a variety of ways, for example, graphically, numerically, or in tables.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- investigating measurement within areas of interest, for example, technology, cultural expressions of art forms, architecture, music, dance, the human body, sport, environmental issues, and exploring the use of a range of measuring devices and scales, for example, the Richter scale, and electronic and non-electronic devices;
- exploring the use of 2-dimensional scales to measure qualitative data such as attitudes, feelings, opinions, and behaviours, and discussing the limitations of this form of measurement;
- developing the understanding needed to be able to solve measurement problems in context;
- consolidating the concepts of area, volume, and mass;
- further exploring the equivalence of units, for example, that 1 millilitre of water, 1 cubic centimetre of water, and 1 gram of water represent the same quantity;
- solving problems in practical contexts involving irregular and composite areas and volumes, and using appropriate models, for example, by using a cylinder to model a tree to estimate the volume of wood, and naming upper and lower limits for the results;
- using investigations to develop formulae for perimeters and areas (rectangles, triangles, parallelograms, trapezia, and circles) and volumes (cuboids and other prisms, cylinders, cones, pyramids, and spheres);
- finding π as the ratio of the circumference to the diameter of a circle; and discovering that shapes with a given perimeter do not necessarily enclose the same area, and vice versa;
- designing and using scale representations of everyday situations, for example, sewing patterns or garage plans.

Developing concepts of time, rate, and change

Students should be:

- using techniques such as graphing and averaging to explore rates and rates of change. For example, tax rates, pollution rates measured in parts per million, acceleration, inflation, stocking rates, and unit pricing.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students write a report on an investigation, using mathematical formulae. For example, do all boxes made from A4 paper, by removing a square from each corner and folding up the edges, hold the same amount? Investigate how the volume of the box varies with the size of the squares cut out, by drawing diagrams, making a table of dimensions, and using a spreadsheet. Use interpolation and successive approximations to find the maximum volume to a reasonable order of precision.

  Using this example, teachers could assess students' ability to:
  - interpret and use information about rates presented in a variety of ways, for example, graphically, numerically, or in tables (M5);
  - record information in ways that are helpful for drawing conclusions and making generalisations (MP5);
  - report the results of mathematical explorations concisely and coherently (MP5).

- Students use formulae for calculating perimeter, area, and volume in practical contexts (rectangles, triangles, parallelograms, trapezia, circles, prisms, pyramids, and spheres). For example, they:
  - find the perimeter and area of a basketball court, hockey field, pathway, piece of land;
  - find the approximate volume of the trunk of a tree, the human body, a pile of sand;
  - estimate and calculate capacities in a practical context such as the volume of the school pool, and then calculate how much chemical is needed to treat it, given the quantity per litre required.

  Using this example, teachers could assess students' ability to:
  - find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5);
  - design and use models to solve measuring problems in practical contexts (M5).

- Students construct a scale diagram of, for example, a wharenui or a house, and use it for appropriate calculations, such as the cost of floor coverings.

  Using this example, teachers could assess students' ability to:
  - find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5);
  - recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension (G5).

- Students collect data on the price and volume of three sizes of a commodity such as shampoo. They determine which size gives the best value by calculating unit prices.

  Using this example, teachers could assess students' ability to:
  - interpret and use information about rates presented in a variety of ways, for example, graphically, numerically, or in tables (M5);
  - write and solve problems involving decimal multiplication and division (N4);
  - record information in ways that are helpful for drawing conclusions and making generalisations (MP5).

- Students model a physical situation and draw conclusions. For example, lizards can hardly move when they are cold. A big lizard and a small lizard are lying on a rock as the sun comes up. Which lizard moves first? Students explore the relationship between surface area and volume by investigating the use of cubes as a model for the situation, and deduce the result. They explain the result to the class and deduce further general results about the ratios of surface area and volume.

  Using this example, teachers could assess students' ability to:
  - find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5);
  - design and use models to solve measuring problems in practical contexts (M5);
  - devise and use problem-solving strategies to explore situations mathematically (MP5);
  - report the results of mathematical investigations concisely and coherently (MP5).

Sample Development Band Activities

- Students determine optimal shapes within given constraints. For example, they:
  - calculate the largest rectangular area that can be enclosed by a given length of wire, then generalise to other shapes;
  - determine the most efficient cross-section shape for spouting;
  - explore ways of minimising cost when producing 375-ml cans.
- Students invent formulae which describe properties (edge length, surface area, volume) of regular solid figures, for example, prisms (including cuboids and cylinders), pyramids of various bases, cones, tetrahedra, and spheres. They explain the derivation of each formula.
- Students explore life on another planet. For example, what are the implications of different gravitational forces for measurement?

Measurement

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- demonstrate the knowledge and skills necessary to plan, implement, and evaluate practical measuring tasks;
- design and use a 2-dimensional scale to represent data.

Developing concepts of rate and change

Within a range of meaningful contexts, students should be able to:

- explain the relationship between the gradient of a graph and the rate of change;
- interpret and use information about rates presented in a variety of ways, for example, graphically, or in tables.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- solving measurement problems, in practical contexts, which involve estimating results, selecting appropriate units, measuring appropriately, using the language of measurement, calculating measurements, evaluating and reporting results, stating levels of accuracy;
- measuring qualitative data such as attitudes, feelings, opinions, and behaviours, using 2-dimensional scales;
- consolidating previous work on measurement, and extending it where appropriate;
- discovering that objects with a given surface area do not necessarily enclose the same volume, and vice versa.

Developing concepts of rate and change

Students should be:

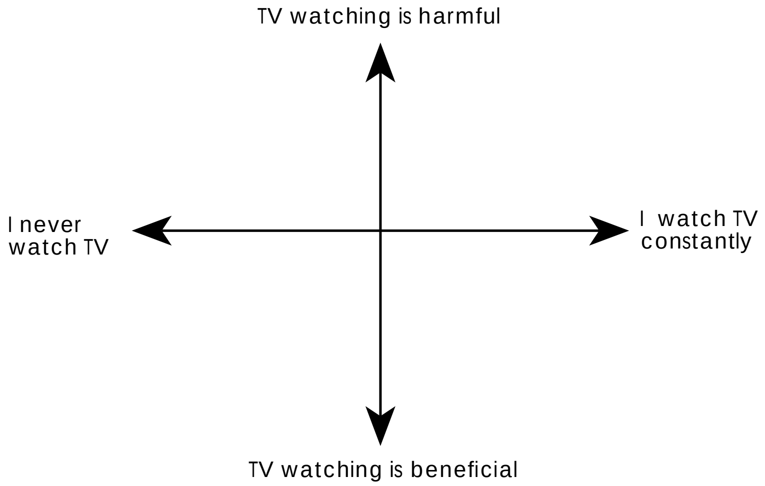
- using, calculating, and interpreting rates in a variety of contexts, for example, retention rates, staffing ratios, and the half-life of nicotine, coffee, alcohol, and radioactive materials;
- establishing the relationship between the gradient of a graph and the rate of change, and interpreting this in context, using technology where appropriate.

Measurement

Sample Assessment Activities

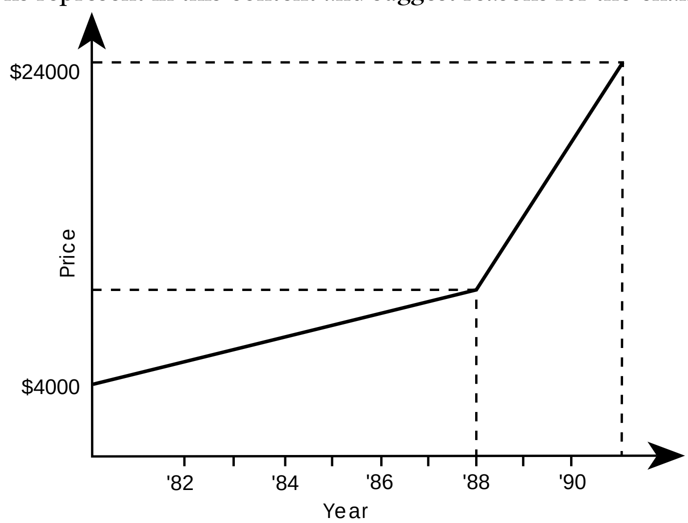
These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students use a 2-dimensional scale of their own design to record and compare qualitative data. For example, they compare the attitudes and behaviours of a sample of secondary school pupils.



Using this example, teachers could assess students' ability to:
- design and use a 2-dimensional scale to represent data (M6);
- record information in ways that are helpful for drawing conclusions and making generalisations (MP6);
- sketch and interpret graphs which represent everyday situations (A5).

- Students interpret a straight-line graph, explaining what the slopes represent in a given context, and suggest reasons for changes in slopes. For example, the diagram shows the graph used by a used-car salesperson to estimate the selling price of a particular brand of car. They explain what the slopes of the straight-line sections represent in this context and suggest reasons for the change in slope.



Using this example, teachers could assess students' ability to:
- explain the relationship between the gradient of a graph and the rate of change (M6);
- interpret and use information about rates presented in a variety of ways, for example, graphically, or in tables (M6);
- interpret information and results in context (MP6);
- graph linear rules and interpret the slope and intercepts on an integer co-ordinate system (A5).

• Students plan, implement, and evaluate a practical task by using known geometrical shapes to model everyday objects. For example, by choosing an appropriate geometric solid to model a pile of sand tipped by a truck, students calculate whether or not there is sufficient sand in a pile to fill a cuboid-shaped sand pit (after taking approximate measurements). Students estimate upper and lower limits for the volumes, and discuss, in general terms, the inaccuracies inherent in their model.

  Using this example, teachers could assess students' ability to:
  - demonstrate the knowledge and skills necessary to plan, implement, and evaluate practical measuring tasks (M6);
  - find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5);
  - devise and use problem-solving strategies to explore situations mathematically (MP6).

Sample Development Band Activities

• By writing their own computer program, or in some other way, students construct a sequence of fractals, such as the Koch (simple snowflake) curve, and investigate factors which determine the perimeter and area of a fractal.

• Students use a computer or calculator to find the area of an interval under a curve by subdividing the area into smaller and smaller sub-intervals, approximating them to trapezia, and adding their areas.

• Students plan nets for, and construct, a number of regular-shaped solids with specified parameters, for example, a regular tetrahedron with a volume of 1 litre.

Measurement and Calculus

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- model a given situation, using trigonometry (including radian measure) to find and interpret measures in context, and evaluate their findings.

Developing concepts of rate and change

Within a range of meaningful contexts, students should be able to:

- sketch the graph of a gradient function from the graph of a function and explain the relationship between them;
- determine and use an expression for the rate of change of a variable and apply it to practical situations such as maximum, minimum, velocity, and acceleration;
- use integration and anti-differentiation in real and simulated situations.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- exploring angle measure, including circular measure, and applying it to practical situations;
- using trigonometric functions to calculate distances, angles, and areas, including the use of sine rule, cosine rule, and area of triangle formula.

Developing concepts of rate and change

Students should be:

- exploring the historical development of calculus and its notations;
- investigating the relationship between a variable and its rate of change through experimental and practical work;
- investigating and interpreting the relationship between a graph and its slope from a rate of change point of view, for example, by using a graphics calculator or suitable graphing package to "zoom in" on a point on a graph until the curve appears to be a straight line, and then find the gradient;
- discussing applications of differentiation, including practical problems on maxima and minima;
- calculating areas under curves by a variety of means such as counting squares, drawing trapezia or rectangles, or by methods of their own choice;
- discovering a relationship between anti-differentiation and integration;
- applying integration, including its use in finding areas under polynomial curves.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students estimate a height or distance not accessible to them. They then take the necessary measurements and calculate the height or distance by using the sine rule or cosine rule.

  Using this example, teachers could assess students' ability to:
  - model a given situation, using trigonometry (including radian measure) to find and interpret measures in context, and evaluate their findings (M7);
  - devise and use problem-solving strategies to explore situations mathematically (MP7);
  - use equipment appropriately when exploring mathematical ideas (MP7);
  - write appropriate equation(s) or inequation(s) to describe a practical situation (A7).

- Students discuss in groups graphs of, for example, the velocities of cars on race tracks, and match the correct graph to each track.

  Using this example, teachers could assess students' ability to:
  - sketch the graph of a gradient function from the graph of a function and explain the relationship between them (M7);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP7);
  - model a variety of situations, using graphs (A7).

- Students use graphing techniques to find the optimum value of a given situation. For example, a sailor wishes to find the optimum position of the sail when sailing across the wind. She experiments with the angle of the boom and notes her speed in a table.

Angle of Boom (° from centre line of the boat)	0	10	20	30	40	50
Speed through the water (m/s)	1	3.2	4.9	5.75	6.25	5.95

  Students graph the data to find the optimum angle for greatest speed.

  Assuming that the curve $y = {}^{-}0.003x^2 + 0.25x + 1$ is a reasonable approximation to the data in the table, the students check by differentiation to find the best angle of the boom.

Using this example, teachers could assess students' ability to:

- determine and use an expression for the rate of change of a variable and apply it to practical situations such as maximum, minimum, velocity, and acceleration (M7);
- interpret information and results in context (MP7);
- use graphical methods to investigate a pattern in data and, where appropriate, identify its algebraic form (A7);
- find by inspection, and interpret, maxima, minima, points of inflexion, asymptotes, and discontinuities for given graphs (A7).

• Students write a report outlining different methods, including integration, of finding approximate areas under a given curve and a curve of their choice, and discuss the advantages and disadvantages of the various methods.

Using this example, teachers could assess students' ability to:

- use integration and anti-differentiation in real and simulated situations (M7).

Sample Development Band Activities

• Students conduct a quantitative exploration of, for example, a local or global environmental issue, and use the information collected to present a report and their conclusions. Students might, for example, investigate the rate of pollution in a river as a function of its distance from the source and its relationship to the densities of animal and plant life in the water.

Measurement and Calculus

Achievement Objectives

Estimating and measuring

Within a range of meaningful contexts, students should be able to:

- compute the cumulative effect of uncertainties of measurement on quantities which are calculated using measured values;
- evaluate uncertainties in a real context and describe their sources, generation, and propagation;
- interpret and evaluate algorithms for practical tasks and calculations.

Developing concepts of rate and change

Within a range of meaningful contexts, students should be able to:

- identify any discontinuity of a given function and find the limit (if it exists);
- choose and apply appropriate differentiation techniques in context;
- identify situations where the rate of change is proportional to the value of the function and use the exponential function in associated problems;
- form a differential equation model of a practical or simulated situation and use it to interpret and draw conclusions about the situation;
- use a variety of methods, including integration, Simpson's rule, and the trapezium rule for finding areas;
- use a variety of methods for finding volumes;
- use a variety of integration techniques to solve real and simulated problems.

Suggested Learning Experiences

Estimating and measuring

Students should be:

- investigating uncertainties arising from practical situations involving measurement and subsequent computation;
- using and constructing a range of algorithms of varying structures for computation, including repetition, summation, and decision making.

Developing concepts of rate and change

Students should be:

- investigating the concepts of continuity and the limit of a function;
- formalising the derivative as $\lim_{h \to 0} \frac{f(x+h) - f(x}{h}$
- differentiating sums, products, quotients, and composite functions (using the chain rule), and finding second and higher order derivatives and functions defined parametrically;
- finding derivatives of trigonometric functions, logarithmic functions, and exponential functions;
- solving real and simulated problems involving variable rates of change, including the use of exponential and logarithmic functions;
- investigating differentiation and integration, including using differential equations to solve real and simulated problems;
- solving differential equations, using a variety of techniques, including separation of variables and numerical methods;
- establishing integration as the limit of a sum and its relationship to anti-differentiation;
- exploring practical situations of summation, including area under a curve and volumes of revolution, using a balance of numerical, non-numerical, calculator, and computer methods;
- integrating polynomial, exponential, trigonometric functions, and rational functions of the type $\frac{ax + b}{cx + d}$ using substitution as appropriate.

Measurement and Calculus

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students investigate the discontinuities of $\frac{x^2 - 4}{x - 2}$ and $\frac{x + 2}{x - 2}$ and evaluate the limits at discontinuities if they exist. State any generalisations about limits at discontinuities on rational functions.

  Using this example, teachers could assess students' ability to:
  - identify any discontinuity of a given function and find the limit (if it exists) (M8);
  - generalise mathematical ideas and conjectures (MP8);
  - use graphical techniques to explore and illustrate $y = x^a$, $y = \frac{ax + b}{cx + d}$, and piece-wise functions (A8);
  - find by inspection, and interpret, maxima, minima, points of inflection, asymptotes, and discontinuities for given graphs (A7).

- Working in groups, students investigate and report on practical problems involving rates of change. For example, a lake contains 106 litres of water, and a factory begins dumping a chemical at the rate of 1 litre per hour. The lake is fed by a fresh stream at 39 litres per hour and keeps a constant level. If the chemical level reaches 0.1%, then the lake is too polluted for drinking.

  Using this example, teachers could assess students' ability to:
  - form a differential equation model of a practical or simulated situation and use it to interpret and draw conclusions about the situation (M8);
  - devise and use problem-solving strategies to explore situations mathematically (MP8);
  - report the results of mathematical investigations concisely and coherently (MP8);
  - choose an appropriate model for real data, including the use of log-log and semi-log techniques, and analyse and interpret the results (A8).

- Students use numerical methods to find the approximate value of $\int \frac{1}{x} dx$ between 1 and other values, and from this show that, for example, the sum of the area from 1 to 2 and the area from 1 to 3 is equal to the area from 1 to 6. They predict similar results.

  Using this example, teachers could assess students' ability to:
  - use a variety of methods, including integration, Simpson's rule, and the trapezium rule for finding areas (M8);
  - make conjectures in a mathematical context (MP8).

- Students investigate and report on the design and construction of a 400-metre all-weather athletics track, including effects of precision of measurement on lane distances, and on required volumes of surface materials.

  Using this example, teachers could assess students' ability to:
  - evaluate uncertainties in a real context and describe their sources, generation, and propagation (M8);
  - interpret information and results in context (MP8);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP8).

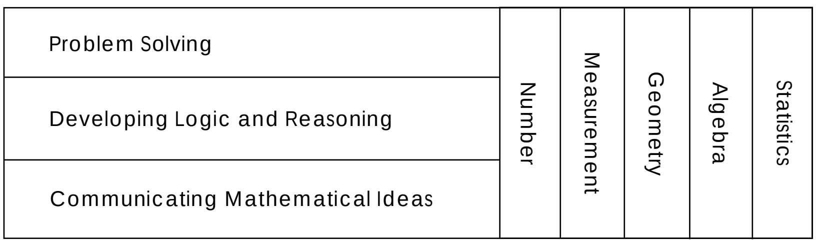
Sample Development Band Activities

- Students develop proofs for standard mathematical results, for example, the limit definition of the derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x}{h}$
- Students find problems which can be modelled appropriately by calculus. For example, they might investigate a range of optimisation problems, such as the ideal distance from a rugby try line from which to take a conversion kick, given that the kick must be in line with the position that the try was scored. Students would have to explain and justify their assumptions.
- Students undertake a project on an application of calculus in economics, demography, physics, or geography. For example, students are given a contour map of a river valley. A dam is to be built at a certain point across the valley. What volume of water can the dam hold? How would the output of the dam be related to the height of the dam? What assumptions about the water catchment, and so on, have been made?
- Students design a tidal power station.
- A model for the spread of an infectious disease assumes that the rate at which the disease spreads through the community is proportional to the product of the number of the people infected and the number of people who have not yet been infected. In a community of 50 000, investigate how many infected people will cause the rate of spread to be greatest.
- Consider taking a medicine by mouth. The drug passes to your stomach and intestines. There, it is gradually absorbed into your bloodstream, which carries it through the body. However, as the blood passes through your kidneys, the drug is removed along with other impurities. So, the drug is absorbed into the body from the intestines at one rate, and removed from the body by the kidneys at another rate. Set up mathematical models of drug concentrations in the form of equations involving rates of change. The problem is to work out what dose is required, and how often it needs to be administered, to maintain the required concentration of the drug in the patient's body.
- Discuss how it is possible for birth rates to remain high for some years even if each mother has few children.

Geometry

The mathematics curriculum intended by this statement will provide opportunities for students to:

- gain a knowledge of geometrical relations in two and three dimensions, and recognise and appreciate their occurrence in the environment;
- develop spatial awareness and the ability to recognise and make use of the geometrical properties and the symmetries of everyday objects;
- develop the ability to use geometrical models as aids to solving practical problems in time and space.



The mathematical processes skills — problem solving, reasoning, and communicating mathematical ideas — are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.

Geometry

Achievement Objectives

Exploring shape and space

Within a range of meaningful contexts, students should be able to:

- identify, and describe in their own language, the following 2-dimensional and 3-dimensional shapes: triangle, square, oblong (non-square rectangle), circle, oval, pentagon, hexagon, diamond, box, cylinder, and sphere;
- classify objects by shape attributes;
- follow and give a sequence of instructions related to movement and position.

Exploring symmetry and transformations

Within a range of meaningful contexts, students should be able to:

- create and talk about symmetrical and repeating patterns;
- rotate their body and other objects through quarter and half turns.

Suggested Learning Experiences

Exploring shape and space

Students should be:

- exploring a variety of shapes and objects by touching, observing, and talking about them;
- sorting, classifying and comparing shapes and objects, using words such as: curved/straight, smooth/rough, flat/bumpy, thick/thin, bottom/top, edge/face/side/corner, and colour names;
- making objects from everyday junk materials and then talking about the shapes used in the construction;
- exploring objects which stack, pack, and roll;
- locating objects and people in space, using words such as:

on	after	inside	under	before	outside
over	beside	middle	underneath	between	behind
forwards	next to	in front of	backwards	on top	along
sidewards	above	around	towards	near	far
away from					

- exploring the construction of simple everyday packages;
- describing the changing appearances of shapes and objects when viewed from different places.

Exploring symmetry and transformations

Students should be:

- experiencing and exploring rolling, spinning, twisting, and turning (quarter and half turns);
- designing and talking about patterns which involve transformations: turning over (reflecting), turning around (rotating), moving without turning (translating), enlarging (for example, writing on a balloon and blowing it up);
- exploring ways of fitting shapes together to cover surfaces (tessellating);
- making and talking about pairs of objects, where one is an enlargement of the other.

Geometry

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- While students are working in pairs:
  - one describes a shape which can be found among a collection of shapes. The other locates it;
  - they find sets of specified objects in a "feely" bag and talk about their similarities and differences;
  - one describes the attributes of an object in a "feely" bag, and the other guesses what it is.

  Using this example, teachers could assess students' ability to:
  - classify objects by shape attributes (G1);
  - classify objects (MP1);
  - record and talk about the results of mathematical exploration (MP1).

- While students are sitting back to back in pairs:
  - one draws a simple picture and describes it in geometric terms. ("I have a large circle. I have two small circles inside that circle near the top.") The partner draws it ;
  - using mosaic shapes, one partner builds a simple model or geometric pattern and describes it. ("I have a red square next to a blue triangle.") The partner makes the model or pattern.

  Using this example, teachers could assess students' ability to:
  - identify, and describe in their own language, the following 2-dimensional and 3-dimensional shapes: triangle, square, oblong (non-square rectangle), circle, oval, pentagon, hexagon, diamond, box, cylinder, and sphere (G1);
  - classify objects (MP1);
  - record and talk about the results of mathematical exploration (MP1).

- Students play "Simon Says", incorporating instructions involving 1, 2, 3, or 4 quarter and half turns;

  Using this example, teachers could assess students' ability to:
  - follow and give a sequence of instructions related to movement and position (G1);
  - rotate their body and other objects through quarter and half turns (G1);
  - find, by practical means, one half and one quarter of a shape, and a half of a set of objects (N1);

- devise and follow a set of instructions to carry out a mathematical activity (MP1).

- Students follow a sequence of three or four instructions (in English or Maori) involving movement, using words such as "towards", "turn", and "beside". For example, they move a toy car around a simple road map.

  Using this example, teachers could assess students' ability to:
  - follow and give a sequence of instructions related to movement and position (G1);
  - devise and follow a set of instructions to carry out a mathematical activity (MP1).

Sample Development Band Activities

- Students seek out, record, and share any symmetrical or repeating patterns they discover throughout a week.
- Students create a sequence of instructions for an original dance, picture, or pattern, try it, and modify it if necessary.
- Students make up sets of instructions to re-create a given dance or pegboard pattern. They test them and, if necessary, modify the instructions.
- Students compare different 2- and then 3-dimensional shapes to find similarities and differences. They record the findings in their own way and share them.
- Having explored tessellating shapes, students experiment with their own created shapes to discover any that tessellate.

Geometry

Achievement Objectives

Exploring shape and space

Within a range of meaningful contexts, students should be able to:

- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects ;
- describe and interpret position, using the language of direction and distance.

Exploring symmetry and transformations

Within a range of meaningful contexts, students should be able to:

- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry;
- make clockwise and anticlockwise turns.

Suggested Learning Experiences

Exploring shape and space

Students should be:

- making, talking about, and classifying shapes and objects from their own and other cultures
  - plane shapes (with paper, geoboards, Logo, and so on)
  - models of spheres, cuboids and other prisms, cylinders, cones, and pyramids (by using playdough, for example);
- exploring tangrams;
- exploring common cardboard boxes and packages and the nets (patterns) used to make them;
- describing the features of objects which have been felt but not seen;
- describing the changing appearances of objects when viewed from different places;
- describing the shape of the cross-section when objects are sliced (carrots, apples, and so on);
- giving and following directions using “clockwise”, “anticlockwise”, “left”, and “right”;
- drawing and reading maps by making use of the four points of the compass (North, South, East, and West);
- describing paths found in the environment, for example, the flight paths of balls thrown in the air, the stream of water from a fountain.

Exploring symmetry and transformations

Students should be:

- exploring and recording the results of turning plane shapes over (reflecting) and around (rotating, both clockwise and anticlockwise);
- exploring and creating geometrical patterns involving translation, reflection, and rotational symmetry;
- exploring ways of covering surfaces with regular shapes (tessellating);
- exploring enlargement of shapes and objects using the overhead projector and by making models, for example, making a model house with playdough.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students play "Simon Says", incorporating instructions involving 1, 2, 3, or 4 quarter and half turns and directions, "clockwise", "anticlockwise", "to the left", and "to the right".

  Using this example, teachers could assess students' ability to:
  - describe and interpret position, using the language of direction and distance (G2);
  - make clockwise and anticlockwise turns (G2);
  - find, by practical means, one half and one quarter of a shape, and a half of a set of objects (N1);
  - devise and follow a set of instructions to carry out a mathematical activity (MP2).

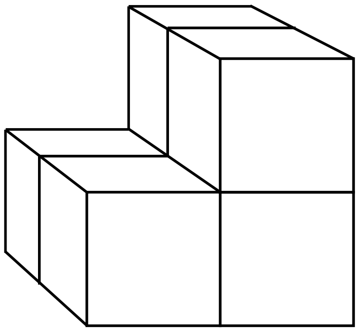
- Students use the language of geometry to describe the distinguishing features of constructions they have made using "junk" material.

  Using this example, teachers could assess students' ability to:
  - make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (G2);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP2).

- Students:
  - rotate an attribute block in a template and explain how many different ways the shape fits;
  - use a shape template attached to a piece of paper by a drawing pin to create a design which has rotational symmetry;
  - identify symmetry in shapes they make while playing whai (string games);
  - identify shapes that have both rotational and reflection symmetry.

  Using this example, teachers could assess students' ability to:
  - create and talk about geometric patterns which repeat (show translation) or which have rotational or reflection symmetry (G2);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP2).

- Students:
  - use blocks to make a model from a picture;



  - make a model from a picture. For example, using playdough and other materials, they furnish a room in a playhouse to match a picture from a magazine.

  Using this example, teachers could assess students' ability to:
  - make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (G2);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP2).

Sample Development Band Activities

- Students use a mirror to create complex designs from simple shapes.
- Students make a wide range of complex origami or paper-folding designs, and investigate the patterns involved. Then, using the information they have found, they create their own design and write instructions for making it.
- Students build a 3-dimensional model scene. They then translate the scene into a series of 2-dimensional sketches and discuss them.
- Using cubes to represent rooms by joining them face-to-face, students try to make as many 4-roomed houses as possible. How many different 5-roomed houses can be made? Investigate rules for knowing whether all possible arrangements have been catered for.

Geometry

Achievement Objectives

Exploring shape and space

Within a range of meaningful contexts, students should be able to:

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry;
- design and make containers to specified requirements;
- model and describe 3-dimensional objects illustrated by diagrams or pictures;
- draw pictures of simple 3-dimensional objects;
- draw and interpret simple scale maps.

Exploring symmetry and transformations

Within a range of meaningful contexts, students should be able to:

- describe patterns in terms of reflection and rotational symmetry, and translations;
- design and make a pattern which involves translation, reflection, or rotation;
- enlarge, on grid paper, simple shapes to a specified scale.

Suggested Learning Experiences

Exploring shape and space

Students should be:

- making simple shapes and objects found in their environment using, for example, paper-folding methods, geoboards, compasses, and Logo;
- making models of objects and scenes represented in photographs and drawings;
- drawing pictures of simple geometric objects;
- talking about simple shapes and objects, using the language of geometry — sides, edges, faces, triangle, rectangle, right angle, parallel, radius;
- exploring tangrams;
- designing simple containers to hold sets of objects such as beans or sweets;
- predicting and checking the shapes of different cross-sections through everyday objects (for example, an ice cream cone);
- giving and following instructions to follow shortest/longest paths between positions (in mazes, maps, Logo);
- drawing and interpreting simple scale maps;
- exploring the idea of locus, for example, the path followed by a small object as it moves on playground equipment, such as a merry-go-round or swing.

Exploring symmetry and transformations

Students should be:

- using mirrors and cutting out shapes defined by creases in folded paper to explore reflection and rotational symmetry;
- designing patterns which involve translation, reflection, or rotation, for example, by using Logo;
- tessellating quadrilaterals, triangles, and regular polygons;
- enlarging or reducing shapes on grid paper, and objects made from cubes, to a specified scale (for example, $\frac{1}{2}$, 2).

Geometry

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

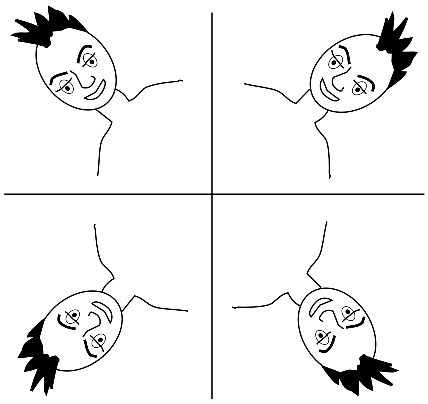
- Students design and make a container to hold, for example, an Easter egg or sweets for sale at a school fair.

  Using this example, teachers could assess students' ability to:
  - design and make containers to specified requirements (G3);
  - devise and use problem-solving strategies to explore situations mathematically (MP3).

- Students make a square by devising and following a series of Logo instructions.

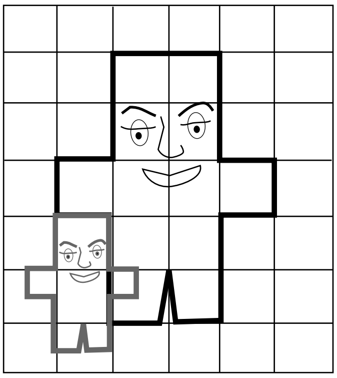
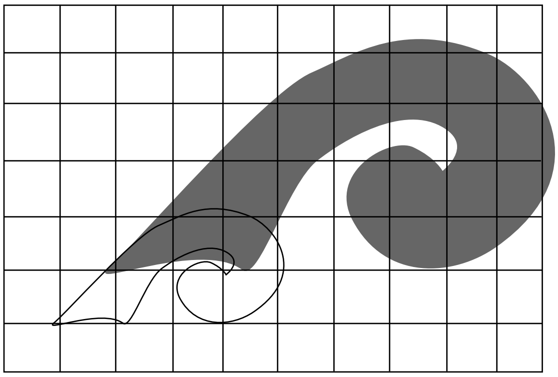
  Using this example, teachers could assess students' ability to:
  - describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (G3);
  - use equipment appropriately when exploring mathematical ideas (MP3);
  - devise and follow a set of instructions to carry out mathematical activity (MP3).

- Students demonstrate translation, reflection, or rotation by making a pattern using a traced image, perhaps a favourite character from a story book.



  Using this example, teachers could assess students' ability to:
  - describe patterns in terms of reflection and rotational symmetry, and translations (G3);
  - design and make a pattern which involves translation, reflection, or rotation (G3);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP3).

- Students enlarge given figures by a specified scale such as 2 or $\frac{1}{2}$. They describe any features that have not changed after enlargement.

Using this example, teachers could assess students' ability to:
- enlarge, on grid paper, simple shapes to a specified scale (G3);
- use equipment appropriately when exploring mathematical ideas (MP3);
- interpret information and results in context (MP3).

Sample Development Band Activities

- Students investigate how devices such as an overhead projector or binoculars use mirrors, or glass prisms as mirrors, in combination. They should then apply their findings to the construction of a mirror-using device, such as a periscope, paying close attention to the precision of the angles involved.
- Students locate and report on a wide range of "matchstick" puzzles, for example, they use 6 matches to form 4 equilateral triangles. They then create original matchstick puzzles.
- Students investigate characteristics of objects that make structures flexible or rigid, fragile or strong, efficient for packing or stacking, and cost effective. For example, they design and construct an efficient supermarket trolley, or bridge, perhaps modelling them by using straws, or newspaper.
- Investigate Escher's tricks with perspective. Make drawings using these ideas.

Geometry

Achievement Objectives

Exploring shape and space

Within a range of meaningful contexts, students should be able to:

- construct triangles and circles, using appropriate drawing instruments;
- design the net and make a simple polyhedron to specified dimensions;
- make a model of a solid object from diagrams which show views from the top, front, side, and back;
- draw diagrams of solid objects made from cubes;
- specify location, using bearings or grid references.

Exploring symmetry and transformations

Within a range of meaningful contexts, students should be able to:

- apply the symmetries of regular polygons;
- describe the reflection or rotational symmetry of a figure or object;
- enlarge and reduce a 2-dimensional shape and identify the invariant properties.

Suggested Learning Experiences

Exploring shape and space

Students should be:

- designing shapes comprising circles, rectangles, triangles, and other polygons, and talking about shapes they make using the language of geometry including words such as sector, arc, perimeter, circumference, semicircle, scalene, equilateral, isosceles, pyramid, vertical, horizontal;
- drawing solids (cuboids and solids made from cuboids) viewed from different positions by first sketching, then using isometric paper, and later squared paper;
- building solids from diagrams which show views from the top, front, back, and side;
- investigating the construction and use of packaging and containers (including regular polyhedra) in the commercial world, for example, to minimise wastage, and for stacking (tessellating end faces);
- estimating, measuring, and drawing angles, including vertically opposite angles, adjacent angles on a straight line, and angles at a point, using a protractor and Logo;
- constructing polygons to given specifications, using paper folding, ruler, compasses and protractor, and Logo;
- using a compass to indicate directions (orienteering, weather directions);
- exploring and using a variety of 2-dimensional grids to specify location (seating plans, maps);
- investigating different pathways, including the shortest path between two locations on a map;
- drawing loci (the paths of objects moving in space) including the flight path of balls, the locus of points which are a fixed distance from a given point (circle), a given straight line (a parallel line), and a circle (a concentric circle).

Exploring symmetry and transformations

Students should be:

- describing the symmetry (reflection and rotational) in patterns, objects, and designs (frieze patterns, kowhaiwhai, wrapping paper, stained-glass window designs);
- exploring the three regular, and the eight semi-regular, tessellations constructed from regular polygons (equilateral triangles, squares, pentagons, hexagons, octagons, and dodecagons);
- investigating the geometry of distortion, using a variety of grids;
- investigating properties of shapes and objects that are or are not changed by enlargement (length, area, volume, angle size, shape, and orientation).

Geometry

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students:
  - use drawing instruments to design a stained glass window or a quilt cover which incorporates squares, triangles, and circles;
  - design a net for, and then construct, a rectangular box to hold a delicate spherical object of known dimensions;
  - design a space-age cycle which makes use of circular and triangular shapes.

  Using this example, teachers could assess students' ability to:
  - construct triangles and circles, using appropriate drawing instruments (G4);
  - design the net and make a simple polyhedron to specified dimensions (G4);
  - use equipment appropriately when exploring mathematical ideas (MP4);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP4).

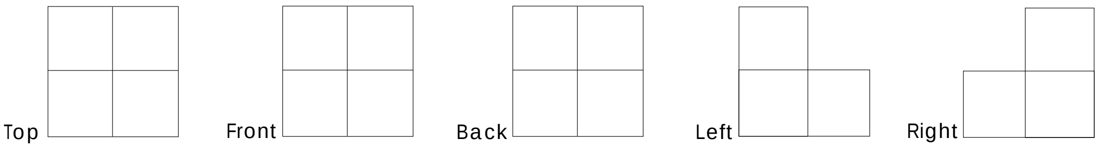
- Students design and make:
  - a tipare (head band), describing any translations or symmetry it possesses;
  - Christmas wrapping paper involving rotational symmetry of order 3, as well as translation;
  - a kowhaiwhai pattern, involving translations and reflections and using, for example, the mangō (hammerhead shark) as the basic motif.

  Using this example, teachers could assess students' ability to:
  - describe the reflection or rotational symmetry of a figure or object (G4);
  - use equipment appropriately when exploring mathematical ideas (MP4);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP4).

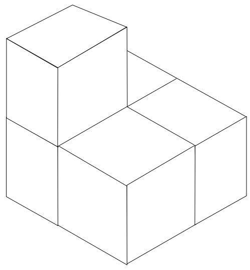
- Students draw all possible pentominoes, decide which ones will fold to give a box without a lid, and then decide which of the pentominoes will tessellate in order that a manufacturer can cut out nets from large sheets of cardboard to ensure minimum wastage.

  Using this example, teachers could assess students' ability to:
  - design the net and make a simple polyhedron to specified dimensions (G4);
  - use equipment appropriately when exploring mathematical ideas (MP4);
  - devise and use problem-solving strategies to explore situations mathematically (MP4);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP4).

- Students use cubes to:
  - construct a 3-dimensional object from drawings of the different faces. For example:



  - build a shape, like the one illustrated below, and then use squared paper to draw top, front, back, and side views.



Using this example, teachers could assess students' ability to:
- make a model of a solid object from diagrams which show views from the top, front, side, and back (G4);
- draw diagrams of solid objects made from cubes (G4);
- use equipment appropriately when exploring mathematical ideas (MP4);
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP4).

- Students working in groups:
  - design and construct a model of the school and grounds, to a scale of 1:200;
  - draw a map of the school grounds and use co-ordinates to find a specified location on the map.

Using this example, teachers could assess students' ability to:
- specify location, using bearings or grid references (G4);
- draw diagrams of solid objects made from cubes (G4);
- enlarge and reduce a 2-dimensional shape and identify the invariant properties (G4);
- devise and use problem-solving strategies to explore situations mathematically (MP4).

Sample Development Band Activities

- Students investigate the "Knight's Tour". For example, is it possible for a knight to visit every square on a chess board once and once only?
- Students design an original board game which requires the use of co-ordinates.
- Students design and construct a detailed scale model of their house.
- Students locate, solve, and explore the mathematics in spatial puzzles such as a Soma cube, or a Greek cross, and use the information they gain to devise an original 3-dimensional puzzle based on a mathematical idea.
- Students research and report on the use of geometrical ideas in art. for example:
  - Escher's use of tessellations;
  - Mitsumasa Anno and "optical illusion";
  - the occurrences of the golden section in art and architecture.
- Construct a kitchen or bathroom from a scale plan. Cost the design.

Geometry

Achievement Objectives

Exploring shape and space

Within a range of meaningful contexts, students should be able to:

- use the angle properties of parallel lines and explain the reasoning involved;
- apply the symmetry and angle properties of polygons;
- use the angle between a tangent and radius property, and the angle-in-a-semicircle property;
- construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors;
- find an unknown side in a right-angled triangle, using scale drawing, Pythagoras' theorem, or an appropriate trigonometric ratio;
- make isometric drawings of 3-dimensional objects built out of blocks;
- solve practical problems which can be modelled, using vectors.

Exploring symmetry and transformations

Within a range of meaningful contexts, students should be able to:

- recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension;
- use the symmetry and angle properties of polygons to solve practical problems;
- use and interpret vectors which describe translations;
- identify and use invariant properties under transformations.

Suggested Learning Experiences

Exploring shape and space

Students should be:

- exploring the relationships between geometry and a range of art forms;
- exploring paper folding and the use of drawing instruments to construct parallel and perpendicular lines, tangents to a circle, using the radius to the point of contact, angle bisectors, mediators, specified angles, and finding centres of circles;
- exploring, talking about, and making the shapes around them, including:
  - traditional Maori patterns, for example, kowhaiwhai, tāniko, tukutuku, whakairo;
  - circles, ellipses, and spirals;
  - precise designs requiring drawing instruments.
- investigating, and applying in practical contexts, Pythagoras' Theorem, and the angle-in-the-semicircle property;
- exploring the properties of polygons, including the relationships between the number of sides of regular and irregular polygons (triangles, quadrilaterals, pentagons, hexagons) and the sum of the interior and exterior angles;
- exploring the relationships between angles (corresponding, alternate, and co-interior angles) when two or more parallel lines are cut by another line;
- further exploring the concept of "locus", for example:
  - the points equidistant from two points, the locus of points equidistant from a point and a line;
  - points equidistant from pairs of vertices and sides in triangles (leading to the circum-circle and in-circle, although these terms are not expected to be known). Compare and contrast with the intersections of medians and altitudes.
- making skeleton models of 3-dimensional objects in order to identify the internal right-angled triangles;
- drawing 3-dimensional objects, using isometric or squared paper.

Exploring symmetry and transformations

Students should be:

- finding, in practical contexts, the scale factors for similar objects in 2 dimensions and 3 dimensions, and calculating unknown measurements;
- enlarging and reducing simple shapes, given the centre of enlargement and scale factor (integral or simple fraction) in 2 and 3 dimensions, and exploring the relationship between the scale factors for length, area, and volume;
- investigating applications of rotation;
- investigating mathematical descriptions of transformations (including vectors to describe translations);
- identifying all the properties of a figure that do and do not change under standard transformations;
- exploring transformations of shapes on non-square grids.

Geometry

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students produce and describe drawings and constructions involving parallel lines. For example, they construct drawings of practical situations such as for diagonal bracing, quilting patterns, or a tipare (headband). They produce a report describing symmetries and indicating angle relationships in the patterns. The report may be presented verbally.

  Using this example, teachers could assess students' ability to:
  - use the angle properties of parallel lines and explain the reasoning involved (G5);
  - construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors (G5);
  - describe the reflection or rotational symmetry of a figure or object (G4);
  - report the results of mathematical explorations concisely and coherently (MP5).

- Students design and draw a model for an object which has a shape based on a regular polygon, for example, a merry-go-round, and discuss the symmetries and angles involved.

  Using this example, teachers could assess students' ability to:
  - construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors (G5);
  - apply the symmetry and angle properties of polygons (G5).

- Students find the centre of a circular piece of material by as many methods as possible, and compare the methods. (A practical context might involve the construction of a model clock.)

  Using this example, teachers could assess students' ability to:
  - use the angle between a tangent and radius property, and the angle-in-a-semicircle property (G5);
  - construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors (G5);
  - describe the reflection or rotational symmetry of a figure or object (G4);
  - devise and use problem-solving strategies to explore situations mathematically (MP5);
  - report the results of mathematical explorations concisely and coherently (MP5).

- Students prepare a poster explaining the construction of the golden rectangle and the subsequent constructions to produce spirals found in nature.

  Using this example, teachers could assess students' ability to:
  - construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors (G5);
  - recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension (G5);
  - devise and follow a set of instructions to carry out mathematical activity (MP5).

- Students design a board game which involves the use of vectors.

  Using this example, teachers could assess students' ability to:
  - solve practical problems which can be modelled using vectors (G5);
  - use and interpret vectors which describe translations (G5);
  - devise and follow a set of instructions to carry out mathematical activity (MP5).

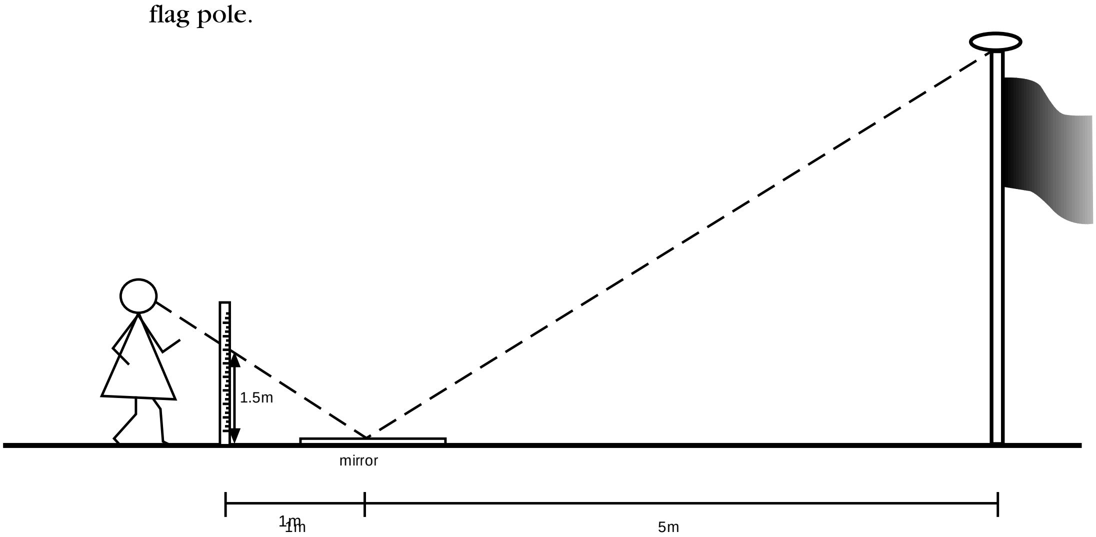
- Students sketch a collection of prisms with irregular bases (for example, with an L-shaped base).

  Using this example, teachers could assess students' ability to:
  - make isometric drawings of 3-dimensional objects built out of blocks (G5).

- Students design a net for, then construct, a box with the shape of a triangular-based pyramid, to hold a delicate spherical object of known dimensions.

  Using this example, teachers could assess students' ability to:
  - design the net and make a simple polyhedron to specified dimensions (G4);
  - make conjectures in a mathematical context (MP5);
  - devise and use problem-solving strategies to explore situations mathematically (MP5).

- Students write a report on the use of a practical measurement which involves similar figures, for example, the method illustrated below, to find the height of a flag pole.



  Using this example, teachers could assess students' ability to:
  - recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension (G5);
  - use the symmetry and angle properties of polygons to solve practical problems (G5);
  - devise and use problem-solving strategies to explore situations mathematically (MP5);
  - use equipment appropriately when exploring mathematical ideas (MP5);
  - design and use models to solve measuring problems in practical contexts (M5).

- Students test conjectures. For example, they might investigate the following proposition: "You cannot draw a circle touching all 3 vertices of a 3, 4, 5 triangle." Students explain their reasoning. They then make a more general conjecture and test it.

  Using this example, teachers could assess students' ability to:
  - make conjectures in a mathematical context (MP5);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP5).

Sample Development Band Activities

- Students investigate the applicability of Pythagoras' Theorem for geometric shapes other than right-angled triangles, and propose and test their own theorems for other regular figures.
- Students investigate and report on applications of geometry in architecture.
- Students present a report on a series of investigations on the theme of inscribing figures within other figures. For example, what is the biggest square that can be drawn within a given triangle? If the area of the triangle is fixed, what shape allows the biggest square? Make, test, and report on conjectures about higher-order shapes within other figures.
- Students investigate other geometries, for example, rubber sheet topology.
- Invent a tool for producing right angles to be used by a clothing manufacturer, draftsperson, or builder.

Geometry

Achievement Objectives

Exploring shape and space

Within a range of meaningful contexts, students should be able to:

- draw and interpret 2-dimensional representations of 3-dimensional objects;
- find unknown angles and lengths in practical problems which can be modelled by triangles, using scale drawing, angle properties of triangles, Pythagoras' theorem, trigonometric ratios, the sine rule, or the cosine rule;
- identify and solve right-angled triangles within 3-dimensional objects and drawings;
- explore and describe a locus formed in a practical situation.

Exploring symmetry and transformations

Within a range of meaningful contexts, students should be able to:

- apply the relationship between the scale factors for length, area, and volume;
- explain the effect of negative scale factors for enlargement;
- describe the net effect of combining two or more transformations (reflections, rotations, or translations).

Suggested Learning Experiences

Exploring shape and space

Students should be:

- calculating unknown lengths and angles in triangles generated in practical problems;
- describing the sections of 3-dimensional objects, exploring the angles and planes found in them, and identifying and using right-angled triangles to calculate dimensions;
- finding measures of circles such as arc lengths, area of sector, and further angle properties in circles, including those associated with chords, and exploring the relationships between them;
- Solving practical problems, using co-ordinate systems on a sphere (latitude and longitude) to extend ideas of circle geometry and trigonometry;
- investigating further loci, for example, methods of drawing conic sections;
- investigating loci which are dependent on frame of reference, for example, the path made by a person walking from the centre to the outside of a moving merry-go-round, the path of a newspaper thrown from a moving car.

Exploring symmetry and transformations

Students should be:

- investigating combinations of two transformations and, where possible, describing the single transformation with the same result;
- finding the centre and angle of rotation on figures involving rotation;
- investigating symmetry transformations of frieze patterns;
- exploring further the scale factors for area, surface area, and volume, using them in practical situations, and exploring negative scale factors;
- exploring the images when simple geometric shapes are rotated or reflected, including the generation of solids of revolution;
- using a calculator or computer to explore transformations of graphs of functions, and of 2-dimensional and 3-dimensional objects.

Geometry

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students construct a 3-dimensional object and calculate its angles and dimension. For example, they construct a right pyramid with a rectangular base 20 cm by 30 cm and a height of 25 cm, and calculate the slant height and the angles the sloping edges and the sloping sides make with the base.

  Using this example, teachers could assess students' ability to:
  - identify and solve right-angled triangles within 3-dimensional objects and drawings (G6);
  - design the net and make a simple polyhedron to specified dimensions (G4);
  - use equipment appropriately when exploring mathematical ideas (MP5).

- Students accurately mark out a scale drawing of a playing court or field, and investigate situations such as horizontal distances traversed by players or balls, or optimum angles for certain plays. For example, how far does the first pass in net-ball travel if the receiver is standing on the intersection of the side line and the "third line"?

  Using this example, teachers could assess students' ability to:
  - construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors (G5);
  - find unknown angles and lengths in practical problems which can be modelled by triangles, using scale drawing, angle properties of triangles, Pythagoras' theorem, trigonometric ratios, the sine rule, or the cosine rule (G6);
  - demonstrate the knowledge and skills necessary to plan, implement, and evaluate practical measuring tasks (M6);
  - design and use a 2-dimensional scale to represent data (M6);
  - pose questions for mathematical exploration (MP6).

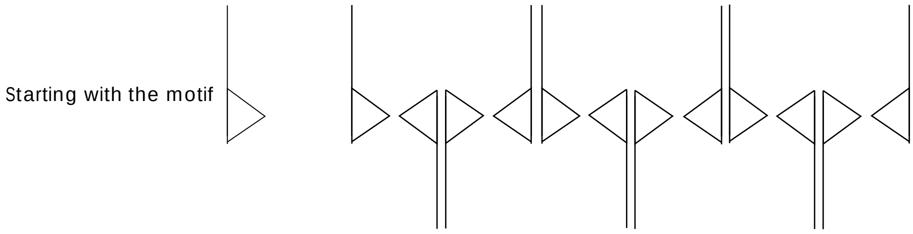
- Students investigate applications relating to length, area, volume, and profit. For example, a baker makes a certain-sized birthday cake for $25 and agrees to make another one, with all the dimensions doubled, for $50. Students might investigate, for example, the amount of icing, the length of frilly paper edging, the number of calories, and the profit.

  Using this example, teachers could assess students' ability to:
  - apply the relationship between the scale factors for length, area, and volume (G6);
  - pose questions for mathematical exploration (MP6);
  - devise a strategy to solve a number problem (N6);
  - estimate and calculate answers, making efficient use of a calculator, where appropriate, as part of solving a problem (N6);
  - find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5).

- Students explain the inversion of the image in a pin-hole camera and specify the scale factor.

  Using this example, teachers could assess students' ability to:
  - explain the effect of negative scale factors for enlargement (G6);
  - recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension (G5).

- Students display results and present conclusions from extended investigations. For example, starting with a simple motif, students draw frieze patterns, using combinations of horizontal and vertical reflections, half turns, and translations. How many different friezes can be created from the same motif? What are their symmetries? What happens when extra transformations such as rotations of 90° and 270° are also allowed? For example, the illustration below shows half-turn and vertical reflection.



  Using this example, teachers could assess students' ability to:
  - describe the net effect of combining two or more transformations (reflections, rotations, or translations) (G6);
  - make conjectures in a mathematical context (MP6);
  - prove or refute mathematical conjectures (MP6);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP6);
  - report the results of mathematical explorations concisely and coherently (MP6).

Sample Development Band Activities

- Students investigate the geometry of the ellipse and report on the occurrence of ellipses in nature, for example, in planetary orbits, and the applications of ellipses in technology and construction, such as in the building of a fale or a whisper chamber.
- Students investigate the classical theorems of Euclidean geometry, and their traditional proofs. They use the theorems as axioms in proving other results.
- Students research the origins and uses of different geographic map projections, such as the Mercator projection. They produce a report on the mathematics underlying such projections, together with a geometrical explanation of the distortions inherent in them.
- Students draw and interpret networks to represent systems such as transport, traffic flow, utility connections, or the body's circulatory system. They construct and interpret route matrices.
- Design a garment from material with a geometrical design and produce a detailed pattern to make it.

Geometry

Achievement Objectives

Exploring co-ordinate geometry

Within a range of meaningful contexts, students should be able to:

- find the distance between 2 points on a co-ordinate plane;
- given an equation representing a straight line in 2 dimensions, find its gradient;
- write and use equations of straight lines in 2 dimensions, given necessary and sufficient information (including a point and the equation of a parallel or perpendicular line);
- find the co-ordinates of the intersection of a straight line and a curve, given their equations;
- convert between rectangular and polar co-ordinates in 2 dimensions.

Exploring networks

Within a range of meaningful contexts, students should be able to:

- choose an appropriate network to organise and visually represent information;
- systematically develop, and critically evaluate, optimal solutions using networks.

Suggested Learning Experiences

Exploring co-ordinate geometry

Students should be:

- using the language and techniques of co-ordinate geometry to investigate, describe, and derive properties of 2- and 3-dimensional shapes (using computer software where appropriate);
- investigating properties of lines, including conditions for parallel and perpendicular lines, and recognising common features associated with the forms $y = mx + c$, $ax + by + c = 0$, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$;
- investigating the intersection of pairs of straight lines and a straight line and a curve (including curves such as a circle with centre (0,0), $y = ax^2$, $y = \frac{a}{x}$, using both algebraic and graphical techniques;
- solving practical problems involving lines, distances, and angles;
- exploring co-ordinate systems, including equivalences between rectangular and polar co-ordinates, using appropriate technology.

Exploring networks

Students should be:

- investigating shortest paths in networks, for example, using a skeleton map of a town to find the shortest path between two places;
- investigating the minimum spanning tree of a network, for example, using a skeleton map of a town to find the minimum length of wire required to connect given places in the town to a telephone service;
- exploring traversability in networks, for example, the postie problem, or the Königsberg bridge problem;
- using networks to model and investigate activities such as simple manufacturing processes, simple project management, and time-management schedules. Appropriate contexts for this study could include preparing meals, or mounting a school drama production.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students apply co-ordinate geometry techniques in mathematically rich contexts. For example:

  - Given the co-ordinates of the vertices of a triangle (or the equations of the three sides), students find the co-ordinates of the circum-centre.

  - Find the centre of mass of a triangular object by modelling the triangle using co-ordinate geometry, and checking answers by experimentation.

  Using this example, teachers could assess students' ability to:
  - find the distance between 2 points on a co-ordinate plane (G7);
  - given an equation representing a straight line in 2 dimensions, find its gradient (G7);
  - write and use equations of straight lines in 2 dimensions given necessary and sufficient information (including a point and the equation of a parallel or perpendicular line) (G7);
  - form and solve linear equations, simultaneous equations… (A6);
  - devise and use problem-solving strategies to explore situations mathematically (MP7).

- Students investigate and report on the application of co-ordinate geometry to the solution of practical problems. For example, investigate and report on trends in finance which are modelled by two lines which intersect.

  Using this example, teachers could assess students' ability to:
  - interpret information and results in context (MP7);
  - find the co-ordinates of the intersection of a straight line and a curve, given their equations (G7).

- Students devise and/or analyse a board game using both polar and rectangular co-ordinates, and report.

  Using this example, teachers could assess students' ability to:
  - convert between rectangular and polar co-ordinates in two dimensions (G7);
  - pose questions for mathematical exploration (MP7);
  - report the results of mathematical investigations concisely and coherently (MP7).

- Students working in groups plan a venture involving network analysis for process planning and optimisation. For example, a class "business" plans to make and distribute pizzas to customers in the town.

  Using this example, teachers could assess students' ability to:
  - choose an appropriate network to organise and visually represent information (G7);
  - systematically develop, and critically evaluate, optimal solutions using networks (G7);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP7);
  - devise and use problem-solving strategies to explore situations mathematically (MP7).

Sample Development Band Activities

- Students investigate other geometries such as geometry on a sphere, cylinder, or rubber sheet (topology).
- Students investigate navigation on the earth's surface.
- Students investigate the use of triangulation in surveying and then, for example, in locating satellites.
- Through a process of conjecture and justification, students find minimum sets of conditions for which two triangles are congruent. They then use congruence conditions for triangles as axioms in order to test the congruence of any two triangles. Students should then investigate conditions for congruence of quadrilaterals and 3-dimensional figures such as tetrahedra.
- Construct geodesic domes.

Geometry

Achievement Objectives

Exploring co-ordinate geometry

Within a range of meaningful contexts, students should be able to:

- describe the nature of conic sections and sketch graphs of them;
- find equations of conic sections from given conditions and find the equations of tangents and normals to them;
- sketch and describe mathematically curves which occur in the environment.

Suggested Learning Experiences

Exploring co-ordinate geometry

Students should be:

- investigating (for example, by using drawing software) the curves formed from conic sections, exploring a variety of ways of drawing them, and investigating their occurrence in society and in nature;
- forming and investigating equations of the circle, ellipse, parabola, and hyperbola, including standard and parametric forms and conditions for tangents and normals;
- experiencing and investigating, for example, by using Logo, other real and artificial curves such as fractals, snowflake curves, envelopes of curves, helixes, spirals, 3-dimensional curves, and others.

Geometry

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students present a report that outlines the occurrence of conic sections, the features of their graphs, and the transformation of one to another, and demonstrate a variety of methods of drawing them.

  Using this example, teachers could assess students' ability to:
  - describe the nature of conic sections and sketch graphs of them (G8);
  - find equations of conic sections from given conditions and find the equations of tangents and normals to them (G8);
  - use and interpret vectors which describe translations (G5);
  - describe the relationship between members of families of curves in terms of transformations (A7);
  - effectively plan mathematical exploration (MP8).

- Students find the equation of the tangent and normal to the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (acos t, bsin t).

  Using this example, teachers could assess students' ability to:
  - find equations of conic sections from given conditions and find the equations of tangents and normals to them (G8);
  - choose and apply appropriate differentiation techniques in context (M8);
  - generalise mathematical ideas and conjectures (MP8).

- Students use CAD software (or other) to explore designs and geometric properties of objects in 3 dimensions.

  Using this example, teachers could assess students' ability to:
  - describe the nature of conic sections and sketch graphs of them (G8);
  - use equipment appropriately when exploring mathematical ideas (MP8);
  - classify objects, numbers, and ideas (MP8).

- Students investigate and report on the generation of fractals or Hilbert curves, using software such as Logo.

  Using this example, teachers could assess students' ability to:
  - sketch and describe mathematically curves which occur in the environment (G8);
  - effectively plan mathematical exploration (MP8);
  - report the results of mathematical investigations concisely and coherently (MP8).

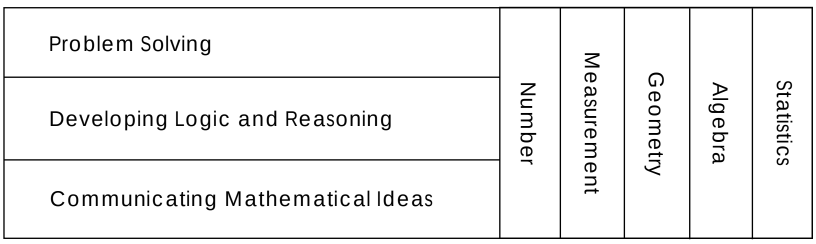
Sample Development Band Activities

- Students explore vector analysis and use vector geometry to prove the classical theorems of Euclidean geometry.
- Students investigate the geometry of shapes used as reflectors. For example:
  - They investigate the light-focusing property of a parabolic curve.
  - They investigate what happens to light emitted from one focus of an ellipse.

Algebra

The mathematics curriculum intended by this statement will provide opportunities for students to:

- recognise patterns and relationships in mathematics and the real world, and be able to generalise from these;
- develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalisations;
- use algebraic expressions confidently to solve practical problems.



The mathematical processes skills — problem solving, reasoning, and communicating mathematical ideas — are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- make and describe repeating and sequential patterns;
- continue a repeating and sequential pattern;
- illustrate and talk about relationships.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

- write number sentences, using = , from story contexts.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- exploring repeating and sequential patterns by making and discussing them, using calculators to generate them, looking at how they continue, and predicting new terms;
- illustrating and talking about relationships, using pictures, arrows, and other methods.

Exploring equations and expressions

Students should be:

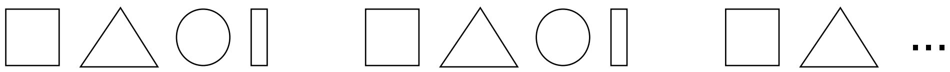
- exploring different ways to show the same number;
- telling number stories involving equals, less than, and greater than; writing and explaining number sentences using =, <, >; and using calculators and concrete material to make statements involving = and the operations (for example, $2 + 3 = 3 + 2$).

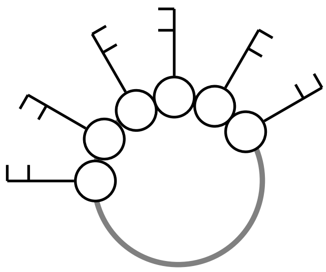
Algebra

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students:
  - create a 4-member repeating pattern with physical objects, and describe where the pattern repeats;
  - continue a 4-member repeating pattern (a pattern in a circle could be included);





  - explain the rules for repeating "people" patterns. For example, who should be next in the pattern: boy, boy, girl, boy, boy, girl, ... and why?
  - create and draw a repeating pattern.

  Using this example, teachers could assess students' ability to:
  - make and describe repeating and sequential patterns (A1);
  - continue a repeating and sequential pattern (A1);
  - devise and use problem-solving strategies to explore situations mathematically (MP1).

- Students illustrate and talk about relationships between two sets, for example, the relationship "likes" between a group of students and their pets (cats, dogs, turtles, canaries).

  Using this example, teachers could assess students' ability to:
  - illustrate and talk about relationships (A1);
  - use their own language … and diagrams to explain mathematical ideas (MP1).

- Students, working in groups, find out and record the number of different ways to get dressed, given four different coloured shirts and three different coloured hats.

  Using this example, teachers could assess students' ability to:
  - illustrate and talk about relationships (A1);
  - devise and use problem-solving strategies to explore situations mathematically (MP1);
  - use their own language … and diagrams to explain mathematical ideas (MP1).

- Students write a number sentence for a story. For example, Genna collected 4 shells from one place on the beach and 2 shells from another place. She has 6 shells altogether.

  Using this example, teachers could assess students' ability to:
  - write number sentences, using = , from story contexts (A1);
  - make up, tell, and record number stories, up to 9, about given objects and sequence pictures (N1).

Sample Development Band Activities

- Students arrange and re-arrange a number of objects to show different ways of representing the number. They read and record each arrangement and explain the result.
- Students make a chart showing the relationships they have sorted from a set of picture cards.
- Students make a number story book and share their entries with others. An entry might be: Mum had three potatoes and Dad had seven. 7+3=10. Dad ate more than Mum. He had 4 more. 7 is 4 more than 3.
- Students explore, record, and describe a variety of number patterns, for example, by using a hundreds board.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- continue a sequential pattern and describe a rule for this;
- use graphs to illustrate relationships.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

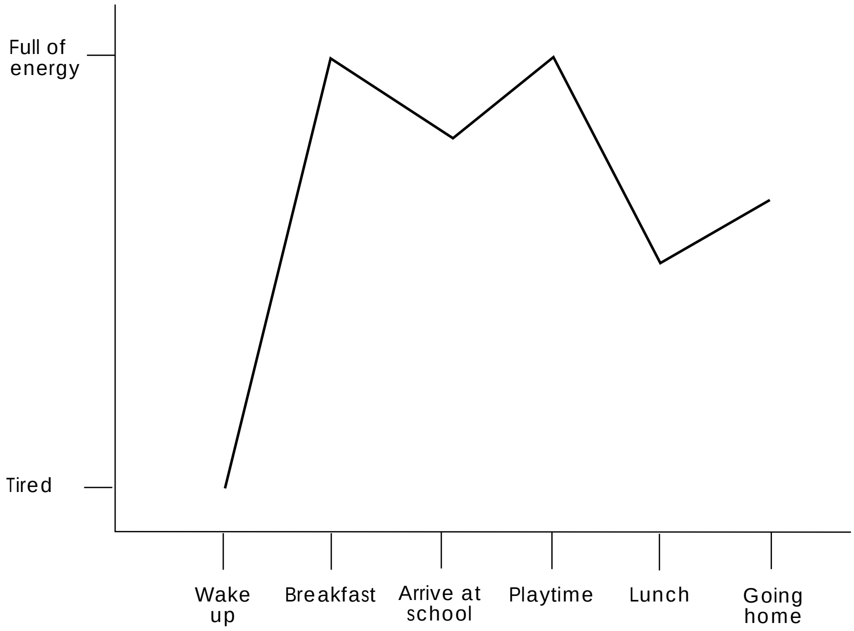
- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than".

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- exploring, creating, describing, and continuing spatial and numerical sequential patterns, using calculators where appropriate, and developing the idea that some patterns continue without end;
- using a variety of diagrams to show relationships, drawn from familiar situations, for example,



- talking about and acting out ideas represented in their own and other graphs.

Exploring equations and expressions

Students should be:

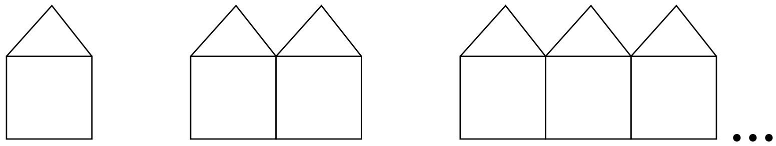
- writing and talking about number sentences using =, <, >;
- using calculators and concrete materials to make statements involving = and the 4 operations, for example, $3 + 4 = 4 + 3$, and $3 \times (4 + 5) = 3 \times 4 + 3 \times 5$ and emphasising that these are alternative ways of expressing the same number;
- exploring the special properties of zero in the context of addition, and 1 in the context of multiplication.

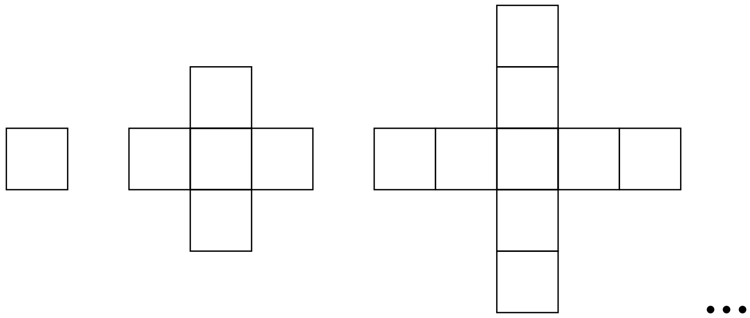
Algebra

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students describe and continue patterns such as:





Using this example, teachers could assess students' ability to:

- continue a sequential pattern and describe a rule for this (A2);
- use words and symbols to describe and continue patterns (MP2).

- Students describe age relationships as number sentences in both directions. For example, "Jo's age plus two years gives John's age" is the same as "John's age minus two years gives Jo's age", and "Jo's age is less than John's age" is the same as "John's age is greater than Jo's age".

Using this example, teachers could assess students' ability to:

- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (A2);
- interpret information and results in context (MP2).

- Students, working in groups, determine and record as many ways as possible of showing a particular number, for example, $16 = 4 \times 4 = 2 \times 2 + 12 = \ldots$

Using this example, teachers could assess students' ability to:

- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (A2).

- Students explain how to get the next numbers in a sequence. For example, what are the next two numbers in the sequence 3, 10, 17 ...? What is the rule?

  Using this example, teachers could assess students' ability to:
  - continue a sequential pattern and describe a rule for this (A2);
  - devise and use problem-solving strategies to explore situations mathematically (MP2).

Sample Development Band Activities

- Students investigate and report on number patterns resulting from operations they have chosen. For example, what happens when numbers are doubled, odd (or even) numbers are added, 4 is added to numbers ending in 7?
- Given sequences of objects, students predict what the 10th, 20th, etc., objects would be. They describe the starting sequence, write their prediction, and explain their reasoning.
- Students explore attribute games. For example:
  - They make a line which has a difference of three attributes between successive blocks.
  - They make lines in which the attribute differences between successive blocks vary according to a pattern.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- describe in words, rules for continuing number and spatial sequential patterns;
- make up and use a rule to create a sequential pattern;
- state the general rule for a set of similar practical problems;
- use graphs to represent number, or informal, relations.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

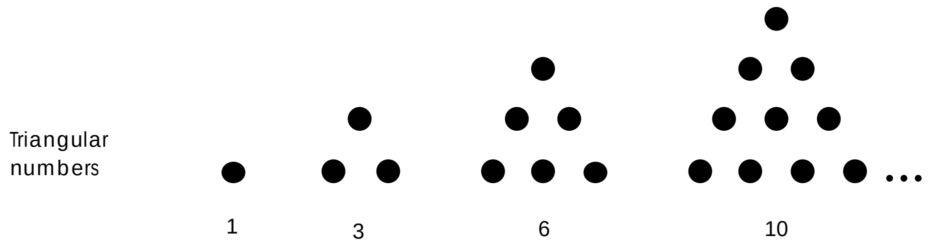
- solve problems of the type $\square + 15 = 39$

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- investigating, creating, and continuing number and spatial sequential patterns, including figurate numbers (triangular, square, and so on), and expressing in words the rules which describe such patterns, using a calculator where appropriate;



- developing an understanding of relations and representing and interpreting them;
- using diagrams to show relationships;
- describing the rule for one- and two-stage "guess my rule" games, and graphing input and output pairs on a number plane, using a calculator where appropriate;
- sketching, interpreting, and writing stories about graphs of familiar situations.

Exploring equations and expressions

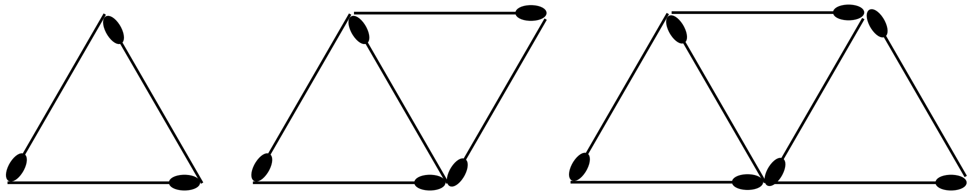
Students should be:

- finding, and explaining in words, simple formulae that can be used to solve a practical problem, for example, buying a number of chocolate bars given the price of one;
- creating and using rules for "think of a number" games, and solving number sentences, for example, $\square + 5 = 12$ and $6 \times \square = 18$
- distinguishing between finite and infinite.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students investigate shape sequences and describe as many rules as they can for making them. For example, matches are used to make triangles. Describe a rule for finding the next member of the sequence.

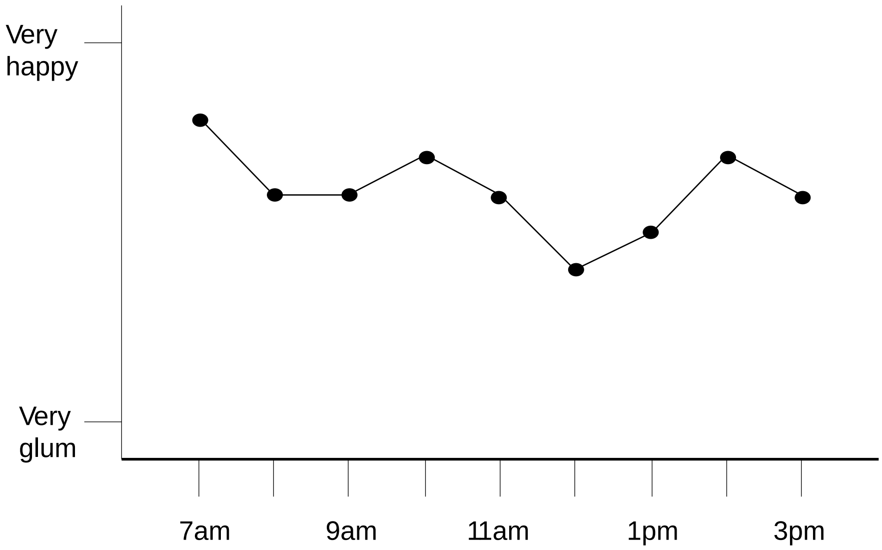


Number of triangles	1	2	3	4	...
Number of matches	3	5	7	9	...

Using this example, teachers could assess students' ability to:

- describe in words rules for continuing number and spatial sequential patterns (A3);
- devise and use problem-solving strategies to explore situations mathematically (MP3).

- Students talk about and sketch a graph showing their moods during the previous day.



Using this example, teachers could assess students' ability to:

- use graphs to represent number, or informal, relations (A3);
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP3).

- Students solve problems of the type: Judy has some marbles. She wins 17 more in a game from her friend Bruce. She then has 35 marbles. How many did she have at the start?

  Using this example, teachers could assess students' ability to:
  - solve problems of the type $\square + 15 = 39$ (A3);
  - write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (N3);
  - devise and use problem-solving strategies to explore situations mathematically (MP3).

Sample Development Band Activities

- Students investigate number patterns, then explore related situations for other patterns. For example:
  - What patterns can be found in the multiplication table of 9? Do similar patterns occur in other multiplication tables?
  - After investigating triangular and square numbers, they explore the possibility of defining 3-dimensional figurate numbers, such as tetrahedral or cubic numbers.
  - Observing that 1+3 = 4, 1+3+5=9, 1+3+5+7=16, they generalise and predict (then test) the sum of the first 10 odd numbers, then the first 20, then the first 200 odd numbers.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- find a rule to describe any member of a number sequence and express it in words;
- use a rule to make predictions;
- sketch and interpret graphs on whole number grids which represent simple everyday situations.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

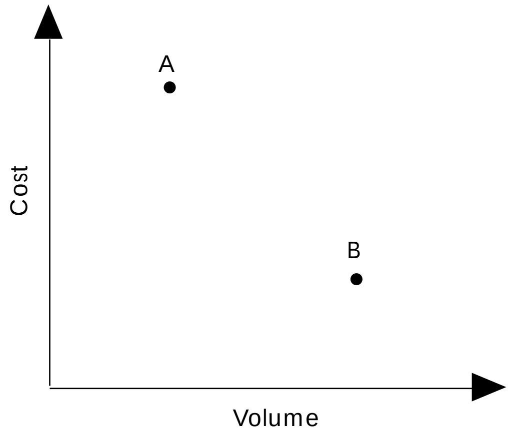
- find and justify a word formula which represents a given practical situation;
- solve simple linear equations such as $2\square + 4 = 16$.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- finding, and continuing, linear number patterns from practical contexts and finding and justifying the rules which describe them;
- developing an understanding of relations and representing and interpreting them;
- sketching graphs which represent familiar situations;
- interpreting a relationship illustrated by points on a graph, for example,



- representing a relationship by a point on a graph;
- writing stories and talking about graphs representing familiar situations;
- generating and graphing sequences from rules expressed in a variety of ways;
- developing strategies for finding rules for linear patterns arising in practical contexts, and using symbols to express these rules.

Exploring equations and expressions

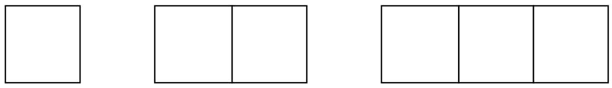
Students should be:

- using, creating, and describing formulae derived from practical contexts, using words and symbols;
- solving number puzzles with whole number solutions which can be represented by simple linear equations such as $2\square + 4 = 16$.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students describe in words suitable rules to represent sequential patterns. For example, suitable descriptions for the following pattern would be "To get the number of lines in any drawing, multiply the number of squares by 3 and add 1." "To get from one value to the next just add 3."

Number of squares	1	2	3	4	
Number of lines	4	7	10	13	...

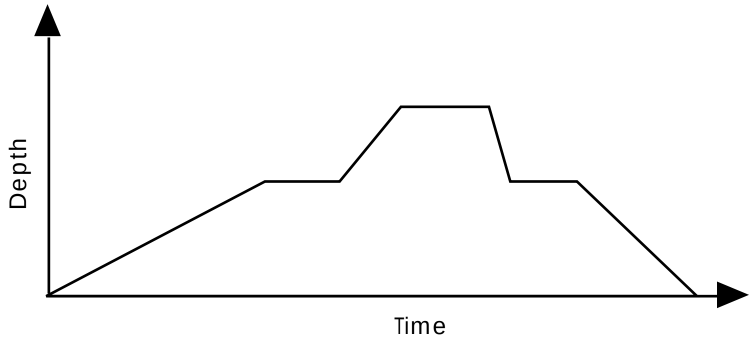
  Predict the number of lines needed for 20 squares, and then graph the sequence.

  Using this example, teachers could assess students' ability to:
  - find a rule to describe any member of a number sequence and express it in words (A4);
  - use a rule to make predictions (A4);
  - devise and use problem-solving strategies to explore situations mathematically (MP4);
  - use words and symbols to describe and generalise patterns (MP4).

- Students:
  - solve informally problems based on relations and explain their reasoning. For example, one band for the school dance charges $150 plus $90 per hour and another charges $60 plus $105 per hour. If $375 has been budgeted for the band, how long would each band play for and which will play longer?
  - find a word formula for problems such as the following: The class is going to the pictures in town. It costs $8.50 per person and $50 for the bus. What is the total cost for the class? What if 5 people are absent?

  Using this example, teachers could assess students' ability to:
  - find and justify a word formula which represents a given practical situation (A4);
  - devise and use problem-solving strategies to explore situations mathematically (MP4);
  - report the results of mathematical explorations concisely and coherently (MP4).

- Students interpret graphical information derived from familiar contexts. For example, the following graph shows the depth of water in a bath. Write a story about the water level as a person takes a bath.

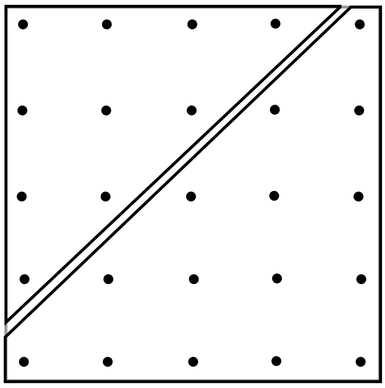


Using this example, teachers could assess students' ability to:
- sketch and interpret graphs on whole number grids which represent simple everyday situations (A4);
- interpret information and results in context (MP4);
- record information in ways that are helpful for drawing conclusions and making generalisations (MP4).

- Students:
  - explore the number patterns found in an array of ordered numbers, such as in a calendar, and investigate the effects of changing the dimensions of the array, for example, a calendar with only 6 days a week;
  - investigate the sum of numbers in opposite corners of any rectangular array of numbers within a calendar.

Using this example, teachers could assess students' ability to:
- find a rule to describe any member of a number sequence and express it in words (A4);
- use a rule to make predictions (A4);
- devise and use problem-solving strategies to explore situations mathematically (MP4);
- report the results of mathematical explorations concisely and coherently (MP4).

- Students investigate the conjecture that "The sum of 2 triangle numbers is always a square number". Students might test the conjecture by trying a range of values, and attempt to prove the conjecture by making use of a geometrical model such as:



  Using this example, teachers could assess students' ability to:
  - find and justify a word formula which represents a given practical situation (A4);
  - interpret information and results in context (MP4);
  - report the results of mathematical explorations concisely and coherently (MP4).

- Students investigate simple strategy games. For example, they find and explain a winning strategy in the following game. Given 2 piles of counters, players take turns to remove at least one counter from any one of the piles. The winner is the person who removes the last counter or group of counters.

  Using this example, teachers could assess students' ability to:
  - use words and symbols to describe and generalise patterns (MP4);
  - make conjectures in a mathematical context (MP4);
  - record information in ways that are helpful for drawing conclusions and making generalisations (MP4).

Sample Development Band Activities

- Students investigate and report on a number of different games of strategy. For example:
  - How many different games of noughts and crosses are possible? They could discuss the concept of isomorphism in this context, and perhaps extend the idea to other contexts.
  - They find a winning strategy for “Nim” or “21 or Bust” on a calculator.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- generate patterns from a structured situation, find a rule for the general term, and express it in words and symbols;
- generate a pattern from a rule;
- sketch and interpret graphs which represent everyday situations;
- graph linear rules and interpret the slope and intercepts on an integer co-ordinate system.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

- evaluate linear expressions by substitution;
- solve linear equations;
- combine like terms in algebraic expressions;
- simplify algebraic fractions;
- factorise and expand algebraic expressions;
- use equations to represent practical situations.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- developing an understanding of relations, and representing and interpreting them;
- interpreting relationships illustrated by points on a graph and representing such relationships in other ways;
- sketching and interpreting graphs which represent everyday situations;
- using rules given in words, symbols, flow charts, or graphs, for example, calculating the dose of medication needed for children of varying ages;
- generating, in practical contexts, linear, quadratic, and other patterns, and finding and justifying the graphs and rules which describe them;
- generating sequences from rules expressed in words and algebraically;
- investigating practical situations that are approximated by linear functions, and investigating the interpretation of the slope and intercept of lines drawn from practical contexts.

Exploring equations and expressions

Students should be:

- talking about the different ways rules can be expressed, for example, $6n + 24 = (n + 4) \times 6 = 6 \times (4 + n)$.
- using algebraic expressions to generalise from numerical instances arising in a practical context, for example, generalising the fact that the diagonal corners of any rectangular array on a calendar have the same sum, and expressing it in algebraic terms;
- developing an ability to solve equations in a problem context by:
  - exploring a variety of strategies for solving equations, for example, trial and improvement, forming patterns, balancing, and reversing flow charts;
  - exploring equations which have either no solution or multiple solutions;
  - representing problem constraints as algebraic inequalities;
- developing confidence in re-arranging and simplifying algebraic expressions by:
  - simplifying expressions by making tables, and by using the "rules of algebra", leading to simplifications such as $x^3 \times x^5 = x^{3+5} = x^8$ and $a^{11} \div a^4 = a^{11-4} = a^7$. Examples might be:

x	$\frac{6x}{2x}$
1	3
2	3
3	3
.	.
.	.

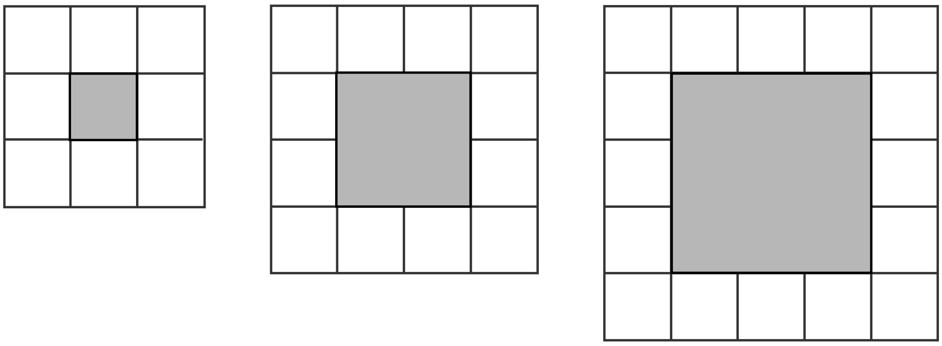
  - re-arranging linear equations and changing the subject of a formula in practical contexts;
  - using number experience, geometric models, and so on, to develop skills in factorising, and expanding them.

Algebra

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students describe relations arising from practical contexts. For example:
  - A square garden is surrounded by square paving stones. What is the relationship between the length of a side of the garden and the number of paving stones needed?



Length of side of garden	1	2	3	4	...
Number of paving stones	8	12	16	20	...

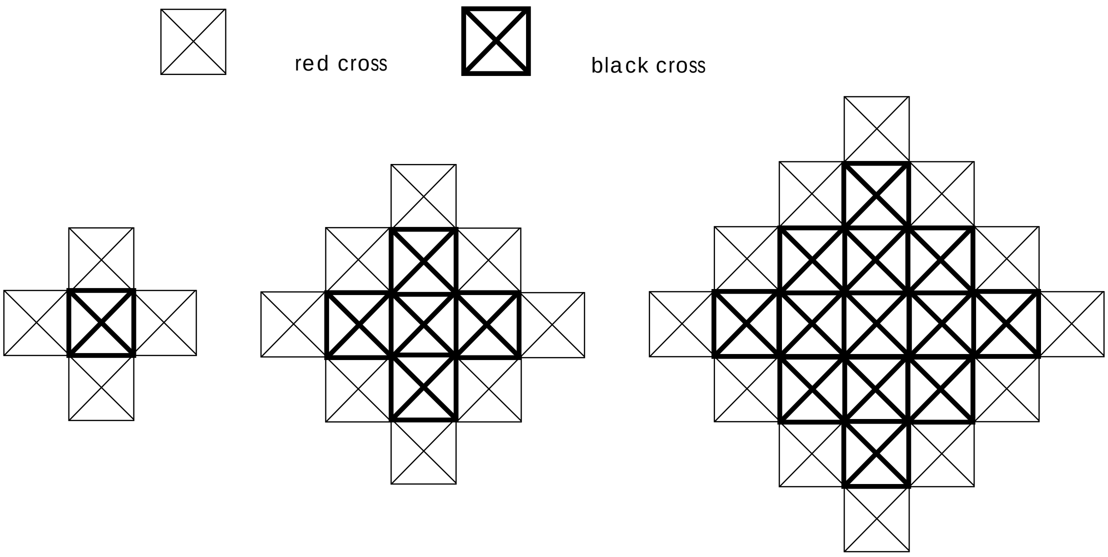
  - Students use pātiki patterns from raranga (weaving) and

    (a) determine the number of squares in each diamond of each colour;

    (b) relate the number of squares to the length of the side;

    (c) investigate and find the rule;

    (d) explain and report the findings.



Using this example, teachers could assess students' ability to:
- generate patterns from a structured situation, find a rule for the general term, and express it in words and symbols (A5);
- use words and symbols to describe and generalise patterns (MP5);
- record information in ways that are helpful for drawing conclusions and making generalisations (MP5).

• Students simplify expressions such as $\frac{3a}{a}$ and $\frac{a^2}{a}$

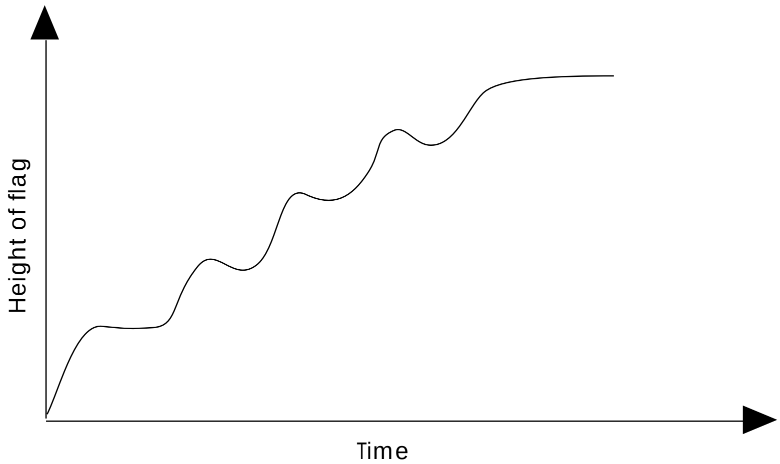
Using this example, teachers could assess students' ability to:
- simplify algebraic fractions (A5).

• Students expand and factorise expressions such as $4y(2y + 5)$ and $3a + a^2$.

Using this example, teachers could assess students' ability to:
- factorise and expand algebraic expressions (A5).

• Students draw and interpret graphs of practical situations in context. For example:

- The graph illustrates a flag being raised on a flag pole. Write a description of the way the flag moves.



- Draw the height vs time graph for a sky diver from the time she leaves the ground in a plane until the time she reaches the ground again with a parachute.

- Students interpret a graph of New Zealand's sheep-meat export earnings over time. Do higher earnings necessarily mean higher prices are being paid?

Using this example, teachers could assess students' ability to:
- sketch and interpret graphs which represent everyday situations (A5);
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP5).

• Students write equations for practical situations and then solve them. For example, the band for the school dance charges \$100 plus \$80 per hour. \$350 has been allocated to pay the band. Students find an equation and solve it to find how long the band would play for.

Using this example, teachers could assess students' ability to:
- solve linear equations (A5);
- use equations to represent practical situations (A5);
- devise and use problem-solving strategies to explore situations mathematically (MP5).

• Students graph equations and interpret the gradient and "intercept" in relation to the equation and/or the practical situation represented. For example, the "Quick Fix" clothing company calculates the cost ($C) of repairs, using the formula C = 10 + 15n where n is the number of hours needed for a job. Find the cost of a repair job taking (a) 3 hours (b) $5\frac{1}{4}$ hours. Graph data for the "Quick Fix" clothing company and interpret the constants 10 and 15 in context. Explain the effect of changing the values of the constants.

  Using this example, teachers could assess students' ability to:
  - graph linear rules and interpret the slope and intercepts on an integer co-ordinate system (A 5);
  - interpret information and results in context (MP5);
  - make conjectures in a mathematical context (MP5).

Sample Development Band Activities

- Students investigate and report on the algebra of 2 x 2 matrices. They could extend the investigation to the applications of 2 x 2 matrices to transformations of plane figures in 2-dimensional co-ordinate geometry, or to the solution of pairs of simultaneous equations.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- form and interpret a graph;
- generate linear and quadratic patterns and find and justify the rule;
- generate a pattern from a rule;
- graph linear, quadratic, and exponential functions, and relations of the form $x^2 + y^2 = r^2$ and $xy = c$

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

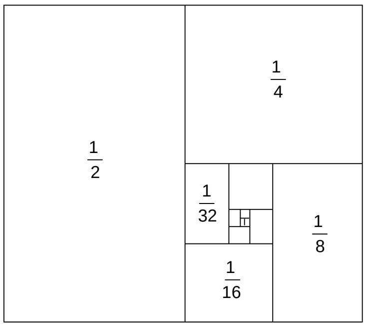
- form and solve linear equations, simultaneous equations, and simple quadratic equations;
- substitute values into formulae.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- generating, in practical contexts, linear, quadratic, and other patterns, and finding and justifying the graphs and rules which describe them;
- using rules expressed in words or symbols to generate sequences;
- using geometrical patterns to investigate infinite numerical patterns, for example, $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 1$;



- using and interpreting formulae arising from practical situations, and sketching and interpreting graphs which illustrate everyday situations;
- investigating practical situations that are approximated by linear, quadratic, exponential, and trigonometric functions (such as the height above the ground of a person riding on a ferris wheel as a function of time);
- investigating families of functions (using a graphics calculator or computer) such as exponential functions and rectangular hyperbolae (including the nature of the inverse relationship);
- describing and interpreting significant features of graphs, for example, maxima, minima, rises, falls, plateaux, periodicity, symmetry, discontinuities, comparison of 2 gradients.

Exploring equations and expressions

Students should be:

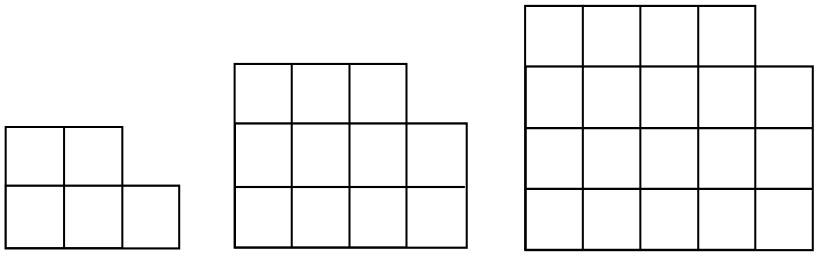
- using algebraic expressions to generalise from numerical instances in practical contexts;
- devising and using strategies for finding rules to represent practical situations, including finding rules for linear and quadratic number patterns by using first and second differences;
- interpreting inequalities and equations arising from practical contexts;
- forming equations (including linear, simultaneous, and quadratic) in practical contexts, and using a range of strategies to solve them, for example, numerical calculator, graphics calculator, computer software, trial and improvement, factorising;
- developing confidence in simplifying and rearranging algebraic expressions by:
  - substituting a range of numerical values into two forms of the same expression to confirm the equivalence;
  - simplifying expressions such as $\frac{x^2 - 4}{x - 2}$.

Algebra

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- *Students find at least two rules for the number sequence, and show by re-arranging that the rules are equivalent.*



5 11 15

Using this example, teachers could assess students' ability to:

- generate linear and quadratic patterns and find and justify the rule (A6);
- substitute values into formulae (A6);
- combine like terms in algebraic expressions (A5);
- factorise and expand algebraic expressions (A5);
- prove or refute mathematical conjectures (MP6);
- record in ways that are helpful for drawing conclusions and making generalisations (MP6).

- *Students substitute in formulae, rearranging if necessary, to find an unknown value. For example:*
  - *In order to calculate the proper dosage of a drug, the following formula is used to calculate a person's body surface area, S m^2, given their mass, W kg, and height, H cm.*

    $S = 0.007184 \times W^{0.425} \times H^{0.725}$

    Sally is 55 kg and is 1.75 m tall. Use the formula to calculate the surface area of her body.
  - *A school group rents a cabin for a cost, \$C, given by* $C=3n+15$ *where n is the number of people in the group. If the cost was \$45 how many people stayed in the cabin?*

Using this example, teachers could assess students' ability to:

- form and solve linear equations, simultaneous equations, and simple quadratic equations (A6);
- substitute values into formulae (A6).

- *Students form simultaneous equations and use both (a) trial and improvement, or number patterns, and (b) elimination or substitution, to solve problems derived from practical contexts. For example: There are 11 competitors in a trike-and-bike race. Altogether there are 29 wheels. How many competitors are on trikes and how many on bikes?*

Using this example, teachers could assess students' ability to:

- form and solve linear equations, simultaneous equations … (A6);
- devise and use problem-solving strategies to explore situations mathematically (MP6).

- Students sketch graphs of given functions or relations. For example:

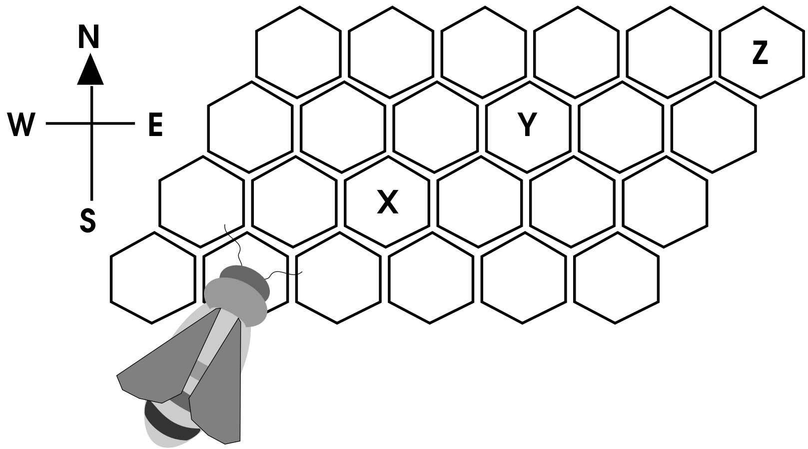
  $y = (2x-3)(x+1)$

  $y = 3x^2$

  $x^2 + y^2 = 25$

  Using this example, teachers could assess students' ability to:
  - graph linear, quadratic, and exponential functions, and relations of the form $x^2 + y^2 = r^2$ and $xy = c$ (A6).
- Students make conjectures about relationships. For example, in geometry, they explore the relationship between angles subtended by arcs at the centre and at the circumference of a circle, and propose a general rule.

  Using this example, teachers could assess students' ability to:
  - make conjectures in a mathematical context (MP6);
  - prove or refute mathematical conjectures (MP6);
  - generate patterns from a structured situation, find a rule for the general term, and express it in words and symbols (A5).
- Working in groups, students devise systematic and efficient strategies to help to deduce results from unfamiliar problems. For example, if a bee in a honeycomb can travel from cell to cell only in the directions east or northeast, how many different paths to a particular cell are possible?



  Using this example,teachers could assess students' ability to:
  - devise and use problem-solving strategies to explore situations mathematically (MP6);
  - record in ways that are helpful for drawing conclusions and making generalisations (MP6).

Sample Development Band Activities

- Students investigate and report on mathematical structures, beginning with groups of order up to 5. The investigation should include the concept of isomorphism, and could include applications of group theory.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- use sequences and series to model real problems and interpret their solutions;
- describe and use arithmetic or geometric sequences or series in common situations;
- model a variety of situations, using graphs;
- sketch graphs and investigate the graph of a function, using a calculator and plotting points if necessary;
- use graphical methods to investigate a pattern in data and, where appropriate, identify its algebraic form;
- find by inspection, and interpret, maxima, minima, points of inflection, asymptotes, and discontinuities for given graphs;
- describe the relationship between members of families of curves in terms of transformations.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

- write appropriate equation(s) or inequation(s) to describe a practical situation;
- choose suitable strategies (graphic, numeric, algebraic, and trigonometric) for finding solutions to equations or inequations, and interpret the results;
- carry out appropriate manipulation and simplification of algebraic expressions.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- investigating geometric and number patterns;
  - using designs from other cultures
  - using numerical approaches
  - modelling
  - using spreadsheets or computer software
- using graphs to illustrate and investigate sequences and series;
- investigating the underlying patterns in everyday situations;
- modelling real and simulated situations;
- exploring the use of algebra to express patterns in sequences and series, including arithmetic and geometric progressions;
- using calculators and computer graphing packages to plot accurately the graph of a general function from a given formula;
- investigating observed or given data to discover relationships;
- exploring the connections (including transformations) between types of graphs and their equations;
  - linear, quadratic, cubic polynomials
  - circles, rectangular hyperbolae
  - exponential functions of the form $y = a^x$
  - logarithmic functions of the form $y = \log_a x$
  - trigonometric functions of the form $y = a \sin(bx+c)$, $y = \tan x$
- investigating families of curves by considering such things as symmetry, periodic behaviour, maxima, minima, behaviour for large values of x and y, discontinuities, and asymptotes.

Exploring equations and expressions

Students should be:

- investigating real and simulated situations using graphical, algebraic, geometric, and technological techniques, including:
  - the manipulation and simplification of algebraic expressions, including rational expressions;
  - the solution of linear and quadratic equations, and pairs of simultaneous equations, one of which may be non-linear;
  - the solution of polynomial equations and the nature of their roots;
  - the solution of trigonometric equations such as $\sin(x+a) = b$, $\cos(ax) = b$;
  - the manipulation of exponents, including fractional and negative exponents;
  - the concept, properties, and manipulation of logarithms.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students find unknown terms and sums in sequences and series derived from practical contexts. For example:
  - The half life of caffeine in the body is three hours. Students model the amount of caffeine in a person's body with a sequence and explore the effects of having successive cups of coffee throughout the day.
  - Students use a strip of paper to investigate the relationship between the number of times the strip is folded in half and the number of creases formed. They find at least two ways of describing the resulting sequence and orally present their investigation. (Does the direction of successive folds make a difference?)

  Using this example, teachers could assess students' ability to:
  - use sequences and series to model real problems and interpret their solutions (A7);
  - effectively plan mathematical exploration (MP7);
  - devise and use problem-solving strategies to explore situations mathematically (MP7);
  - make conjectures in a mathematical context (MP7);
  - use words and symbols to describe and generalise patterns (MP7).
- Students form and apply an algebraic relation for data arising from an experiment. For example, given a table showing distance and time for a ball rolling down a smooth ramp, students write an equation for the distance-time relation and use it to calculate further distances or times.

  Using this example, teachers could assess students' ability to:
  - sketch graphs and investigate the graph of a function, using a calculator and plotting points if necessary (A7);
  - use graphical methods to investigate a pattern in data and, where appropriate, identify its algebraic form (A7).

- Students write and solve simultaneous equations involving at least one non-linear equation, arising from practical situations. For example, write a pair of equations for the amount each employee receives, and find the number of workers employed and the amount each gets, from a company which budgets to pay all employees equally from a total of $10 000. Each employee is paid a retainer of $300 plus an amount equal to four times the number of employees.

  Using this example, teachers could assess students' ability to:
  - write appropriate equation(s) or inequation(s) to describe a practical situation (A7);
  - choose suitable strategies (graphic, numeric, algebraic, and trigonometric) for finding solutions to equations or inequations, and interpret the results (A7).

- Students manipulate and simplify expressions such as:

  $$\frac{x^2 + 2x + 1}{x^2 - 1}$$

  $$\frac{n}{n + 1} - \frac{3}{n - 2}$$

  Using this example, teachers could assess students' ability to:
  - carry out appropriate manipulation and simplification of algebraic expressions (A7).

- Students use their own methods to find the zeros of equations such as $y = 5x^3 - 12x^2 - 16x + 8$ and discuss the nature of the roots. (Satisfactory methods could include a table-building program or graphing software to isolate the roots between pairs of integers, or a successive approximation method, or a "zoom in" function on a calculator or computer.)

  Using this example, teachers could assess students' ability to:
  - choose suitable strategies (graphic, numeric, algebraic, and trigonometric) for finding solutions to equation(s) or inequation(s), and interpret the results (A7);
  - effectively plan mathematical exploration (MP7);
  - devise and use problem-solving strategies to explore situations mathematically (MP7).

- Students develop mathematical models to describe familiar practical situations. For example, a ferris wheel has a radius of 8 m and makes a complete revolution in 12 seconds. Students develop a mathematical model that describes the relationship between the height, h, of a rider above the bottom of the ferris wheel (1 m above the ground) and time, t, seconds.

  Using this example, teachers could assess students' ability to:
  - model a variety of situations, using graphs (A7);
  - write appropriate equation(s) or inequation(s) to describe a practical situation (A7);
  - devise and use problem-solving strategies to explore situations mathematically (MP7);
  - record in ways that are helpful for drawing conclusions and making generalisations (MP7).

- Using a graphing package, or otherwise, students identify significant features of a graph and explain them.

  Using this example, teachers could assess students' ability to:
  - find by inspection, and interpret, maxima, minima, points of inflection, asymptotes, and discontinuities for given graphs (A7);
  - sketch graphs and investigate the graph of a function ... (A7);
  - use equipment appropriately when exploring mathematical ideas (MP7).

- Students present proofs and/or analyse other students' early attempts at proving a proposition, and identify weaknesses in the arguments. For example:

  - They prove that for any number x, $6x - 10 - x^2$ is always negative.
  - They prove or refute the statement, "There is only one value of n for which the positive integers n, n + 2, and n + 4 are all prime."
  - They prove or refute that $n^2 - n + 41$ is prime for all $n \in W$.

  Using this example, teachers could assess students' ability to:
  - substitute values into formulae (A6);
  - generalise mathematical ideas and conjectures (MP7);
  - prove or refute mathematical conjectures (MP7).

Sample Development Band Activities

- Students research and report on paradoxes in algebra, number, statistics, geometry, and logic.
- Students investigate practical situations where mathematical modelling is used and report on the model used, its usefulness, and limitations. The investigation could proceed from an examination of a simple example, such as the proposed use of a Fibonacci sequence to model the growth of a rabbit population.

Algebra

Achievement Objectives

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- use sequences and series to model real or simulated situations and interpret the findings;
- investigate and interpret convergence of sequences and series;
- choose and carry out appropriate manipulation and graphical representation of complex numbers;
- model real and simulated situations, using linear programming techniques, and obtain and interpret optimal solutions;
- use graphical techniques to explore and illustrate $y = x^a$, $y = \frac{ax + b}{cx + d}$ and piece-wise functions;
- choose an appropriate model for real data, including the use of log-log and semi-log techniques, and analyse and interpret the results;
- sketch the graphs of inverse and/or reciprocal functions and explain relationships between them and the original functions.

Exploring equations and expressions

Within a range of meaningful contexts, students should be able to:

- investigate and find numbers of arrangements and selections from a number of objects;
- expand and use binomial expressions for small positive integral exponents;
- use simultaneous equations to model real and simulated situations, and interpret their solutions in a given context;
- use appropriate numerical methods and technology to solve non-linear equations;
- use and prove the factor and remainder theorems;
- solve any quadratic equation and equations of the form $z^n = a$ (for n a positive integer) and $z = r \operatorname{cis}ø$
- carry out the manipulation necessary to use trigonometric expressions in other areas of mathematics;
- find solutions, including the general solution, for trigonometric equations.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- using graphing, modelling, and calculator techniques to explore a range of divergent, convergent, and oscillating sequences and series (including exponential and logarithmic series);
- investigating graphing, using sketching, plotting, computers, and calculators;
- modelling practical situations from science, commerce, and the social sciences;
- using piece-wise functions arising from practical contexts;
- using graphical and algebraic techniques to explore and illustrate reciprocal and inverse functions and the relationships between them, including exponential, logarithmic, and trigonometric functions.

Exploring equations and expressions

Students should be:

- making selections and arrangements leading to permutations and combinations and the binomial theorem for positive integral index;
- investigating and modelling real and simulated situations using graphical, algebraic, geometrical, and technological techniques, including:
  - the concept of 3-space related to 3 dimensions;
  - considering the consistency and uniqueness of solutions, including 2×2 and 3×3 simultaneous equations;
  - the use and proof of remainder and factor theorems;
- exploring the extension of the number system to include complex numbers;
- working with complex numbers in the form $a + ib$ and in polar form to solve quadratic and other equations, including those in the form $z^n = a$ (for n a positive integer) and $z = r\ \text{cis}\,ø$, and using De Moivre’s Theorem;
- exploring relationships between trigonometric expressions and situations that give rise to them.

Algebra

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Using a calculator (or a computer program), students investigate the convergence of the sequences such as $f(n) = \frac{1}{n}$ and $g(n) = (1 + \frac{1}{n})^n$ and estimate values of limits where they exist. Other suitable sequences could be:

  $$\frac{n}{n+1}, \quad \frac{1}{n^2}, \quad \frac{1}{n^3}, \quad \frac{5n}{2n+1}, \quad \frac{1}{n!}$$

  Using this example, teachers could assess students' ability to:
  - investigate and interpret convergence of sequences and series (A8).

- Students calculate possible permutations and combinations in context. For example, typical car registration plate numbers are A 657 HDC (Britain), MPQ 049 (Victoria), UN 5367 (New Zealand). How many unique licence plate numbers are possible in the various states?

  Using this example, teachers could assess students' ability to:
  - investigate and find numbers of arrangements and selections from a number of objects (A8).

- Students solve simultaneous equations in 3 variables by elimination. For example, they:
  - solve by elimination

    $$x + \frac{y}{2} + \frac{z}{3} = 1$$
    $$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 0.75$$
    $$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 0.55$$

  - solve exactly by elimination

    $$x + 0.5y + 0.33z = 1$$
    $$0.5x + 0.33y + 0.25z = 0.75$$
    $$0.33x + 0.25y + 0.2z = 0.55$$

  - explain the differences in the solutions to the two sets of equations.

  Using this example, teachers could assess students' ability to:
  - use simultaneous equations to model real and simulated situations, and interpret their solutions in a given context (A8);
  - interpret information and results in context (MP8);
  - make conjectures in a mathematical context (MP8);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP8).

- Students write a report that shows the need for numerical techniques (both bi-section and Newton-Raphson methods) for solving equations, including a visual description of the methods, and the effect of different starting values. Students solve a given equation by numerical methods and explore a function of their own choosing.

  Using this example, teachers could assess students' ability to:
  - use appropriate numerical methods and technology to solve non-linear equations (A8);
  - use their own language, and mathematical language and diagrams, to explain mathematical ideas (MP8);
  - report the results of mathematical investigations concisely and coherently (MP8).
- Students use a calculator or graphics package and the remainder theorem to identify factors or to locate intervals containing roots to an equation.

  Using this example, teachers could assess students' ability to:
  - use and prove the factor and remainder theorems (A8);
  - use equipment appropriately when exploring mathematical ideas (MP8).
- Students find the real and complex solutions of equations of the form $(x - 1)(x^2 - 2x + 4) = 0$.

  Using this example, teachers could assess students' ability to:
  - solve any quadratic equation and equations of the form $z^n = a$ (for n a positive integer) and $z = r \text{ cis}ø$ (A8).
- Students sketch and interpret the graph of, for example, $y = \tan^{-1}x$.

  Using this example, teachers could assess students' ability to:
  - sketch the graphs of inverse and/or reciprocal functions and explain relationships between them and the original functions (A8).
- Students make conjectures and prove or refute generalisations. For example:
  - They investigate and make conjectures about numbers which are the sum of 3 consecutive whole numbers, 4 consecutive whole numbers, 5 consecutive whole numbers, and so on.
  - They show that for all natural numbers n, 3 is a factor of $n^3 + 2n$.
  - They prove previously unseen results and some standard results (Remainder Theorem, De Moivre's Theorem, Binomial Theorem, alternative formulae for the standard deviation, compound angle formulae, the rule for differentiating composite functions, ${}^nC_r = {}^nC_{n-r}$ and so on).

  Using this example, teachers could assess students' ability to:
  - investigate and find numbers of arrangements and selections from a number of objects (A8);
  - generalise mathematical ideas and conjectures (MP8).

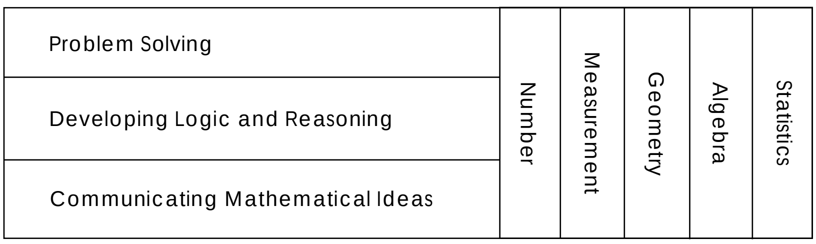
Sample Development Band Activities

- Students investigate the algebra involved in special relativity, including limits as $v \rightarrow c$, for example, in the Lorentz-Fitzgerald contraction.
- Students investigate systems of formal logic — Aristotelian, Boolean, modern.

Statistics

The mathematics curriculum intended by this statement will provide opportunities for students to:

- recognise appropriate statistical data for collection, and develop the skills of collecting, organising, and analysing data, and presenting reports and summaries;
- interpret data presented in charts, tables, and graphs of various kinds;
- develop the ability to estimate probabilities and to use probabilities for prediction.



The mathematical processes skills — problem solving, reasoning, and communicating mathematical ideas — are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra, and statistics.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- collect everyday objects, sort them into categories, count the number of objects in each category, and display and discuss the results.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- classify events from their experiences as certain, possible, or impossible.

Suggested Learning Experiences

Statistical investigations

Students should be:

- working with category variables, for example, kinds of shoes, colours of clothing, types of material;
- talking about experiences and situations such as aspects of their environment, their families, their friends, which can be investigated statistically;
- collecting data and displaying it in ways that tell a story;
- talking about their displays, for example, saying which of the shell types occurred most often in a container full of shells collected from a beach.

Exploring probability

Students should be:

- talking about the likely occurrence of familiar events when using their own language. For example, they should be using language such as will, won't, and might when talking about the likelihood of tomorrow being a sunny day; of it raining on Saturday; of the sun rising tomorrow; of their house being in the same street next week.

Statistics

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students discuss questions such as, "Why do people wear shoes?" "Do they wear the same shoes in winter as they do in summer?" "What shoes do we wear?" The students, working as a class, might choose to check the last question. They make a display, using their own shoes, then draw a picture graph of shoe types. They then discuss their graph.

  Using this example, teachers could assess students' ability to:
  - collect everyday objects, sort them into categories, count the number of objects in each category, and display and discuss the results (S1);
  - pose questions for mathematical exploration (MP1);
  - classify objects (MP1);
  - interpret information in context (MP1).

- Students use the language of certainty, possibility, and impossibility to make statements such as, "Dad might give me cheese sandwiches tomorrow." "This pencil can't be a crayon tomorrow."

  Using this example, teachers could assess students' ability to:
  - classify events from their experiences as certain, possible, or impossible (S1);
  - use their own language … and diagrams to explain mathematical ideas (MP1).

Sample Development Band Activities

- Students devise a maze or a road map with a given number of pathways from one point to another.
- Students could collect data related to an everyday classroom activity, for example, who is ordering what for lunch today. They could translate the data onto a pictograph and interpret the results for classroom action.
- Students make or write a large number of statements that could be classified according to probability — might, can't, always, never, and sometimes. They invite others in the class to classify the statements, and discuss their classifications.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- collect and display category data and whole number data in pictograms, tally charts, and bar charts, as appropriate.

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- talk about the features of their own data displays;
- make sensible statements about the situation represented by a statistical data display drawn by others.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- compare familiar or imaginary, but related, events and order them on a scale from least likely to most likely.

Suggested Learning Experiences

Statistical investigations

Students should be:

- working with objects or whole number variables (for example, ages of children in years);
- talking about situations they have experienced and posing questions for investigation;
- collecting data in a planned way;
- devising ways of displaying their data to show its significant features, and using conventional displays such as pictographs, block graphs, bar graphs, stem-and-leaf graphs, and tally charts.

Interpreting statistical reports

Students should be:

- talking about and interpreting the displays, for example, interpreting the difference between the smallest and the largest values, and explaining in their own language the meaning of clusters or holes in the data display;
- comparing the findings with the expected results;
- reporting to others;
- looking at and talking about the ways others present data.

Exploring probability

Students should be:

- developing a concept of chance, for example, ordering statements and pictures related to their experiences, using words such as certain, likely, unlikely, impossible, and possible;
- exploring ways of finding all possible outcomes of simple events by using concrete materials or pictures of objects.

Statistics

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students pose questions related to a topic, decide which to investigate, and work in groups to collect and display data on a stem-and-leaf graph. For example, if the topic is age, the students may pose the questions "How old are our grandparents?" "How old are our parents?"

  Using this example, teachers could assess students' ability to:
  - collect and display category data and whole number data in pictograms, tally charts, and bar charts, as appropriate (S2);
  - pose questions for mathematical exploration (MP2);
  - record, in an organised way, and talk about the results of mathematical exploration (MP2).

- Students collect the birth dates (day of the month) of the class, and record the data in a stem-and-leaf graph. Supposing that the data collected in the investigation is as below, students talk about the features of the data displayed, such as the most common number, and the spread of numbers.

3	0								
2	3	4	4	5	6	9			
1	1	1	2	4	5	5	7	9	9
0	1	4	7	7	8				

  Using this example, teachers could assess students' ability to:
  - interpret information and results in context (MP2);
  - talk about the features of their own data displays (S2);
  - explain the meaning of the digits in 2- or 3-digit whole numbers (N2).

- Working in pairs or groups, students make up situations about themselves, and rank them from least likely to most likely, for example, "Tomorrow I will be a grown-up." "Tomorrow I will still be a child." "Tomorrow it will rain."

  Using this example, teachers could assess students' ability to:
  - compare familiar or imaginary, but related, events and order them on a scale from least likely to most likely (S2).

Sample Development Band Activities

- Students explore possible combinations, for example, they select 3 different items and state 2 different possible classifications for each. They then count and record the total possible combinations of items and classifications. Students could repeat the activity with a greater number of items and classifications.
- Students collect as many different graphs, charts, and tables as they can from print media, and present the information shown by each in another way. They comment on the advantage or disadvantage of the original presentation.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- plan a statistical investigation of an assertion about a situation;
- collect and display discrete numeric data in stem-and-leaf graphs, dot plots, and strip graphs, as appropriate.

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- use their own language to talk about the distinctive features, such as outliers and clusters, in their own and others' data displays;
- make sensible statements about an assertion on the basis of the evidence of a statistical investigation.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- use a systematic approach to count a set of possible outcomes;
- predict the likelihood of outcomes on the basis of a set of observations.

Suggested Learning Experiences

Statistical investigations

Students should be:

- talking about situations they have experienced, posing questions for investigation, and presenting a plan for a statistical experiment (which could include the use of computers);
- collecting and presenting data in a variety of ways, including strip graphs, dot plots, bar graphs, picture graphs, stem-and-leaf graphs, and tally charts;
- talking about and considering the features of displays of number data, including the overall shape of the data distribution, clusters, middle, and spread;
- organising measurement data to the nearest whole number, using informal approaches and displaying such data;
- reporting on statistical investigations, using pictures and words;
- collecting and graphing simple time-series data such as the height of a classroom-grown bean plant at midday each day.

Interpreting statistical reports

Students should be:

- considering data displays, and making sensible statements about the significant features of the display, and making informal inferences based on them;
- evaluating statements and assertions about a situation represented in a data display;
- developing an awareness of the ways in which statistical information can be presented.

Exploring probability

Students should be:

- developing a concept of probability by investigating games and activities which involve chance, using computers where appropriate;
- using language associated with chance, for example, certain, good chance, even chance, poor chance, impossible;
- considering frequency tables, drawing informal inferences about the populations, for example, “three out of ten cars are white”;
- finding all possible outcomes of an event, using systematic approaches including tree diagrams.

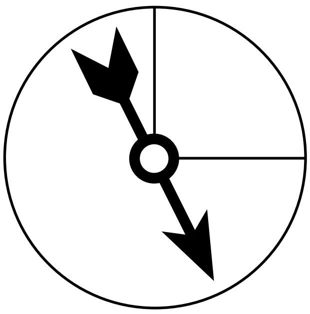
Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students consider an assertion, for example, boys' handspans are larger than girls' handspans. The students must devise a plan to measure and record the handspans of boys and girls in their class, then display and talk about the distinctive features of their data, and draw conclusions relating to the assertion.

  Using this example, teachers could assess students' ability to:
  - plan a statistical investigation of an assertion about a situation (S3);
  - collect and display discrete numeric data in stem-and-leaf graphs, dot plots, and strip graphs as appropriate (S3);
  - use their own language to talk about the distinctive features, such as outliers and clusters, in their own and others' data displays (S3);
  - make sensible statements about an assertion on the basis of the evidence of a statistical investigation (S3);
  - pose questions for mathematical exploration (MP3);
  - interpret information and results in context (MP3).

- Students conduct an experiment and discuss the outcomes. For example, the spinner below is to be spun 100 times. Predict how many times it will land on red.



  Using this example, teachers could assess students' ability to:
  - use a systematic approach to count a set of possible outcomes (S3);
  - predict the likelihood of outcomes on the basis of a set of observations (S3);
  - pose questions for mathematical exploration (MP3);
  - record, in an organised way, and talk about the results of mathematical exploration (MP3).

- Students solve combination problems and explain the method that they have used. For example, Telesia has 4 T shirts and 3 pairs of shorts. How many different outfits can she wear?

  Using this example, teachers could assess students' ability to:
  - use a systematic approach to count a set of possible outcomes (S3);
  - devise and use problem-solving strategies to explore situations mathematically (MP3);
  - use their own language and the language of mathematics and diagrams to explain mathematical ideas (MP3).

Sample Development Band Activities

- Students plan and implement a comprehensive survey. For example, what are the lunch-buying habits of students at the school? They develop a survey form, select a survey sample, collect data, and analyse the results. They formulate a proposal for the principal or board of trustees as a result of the survey.
- Students investigate probabilities in everyday games, such as board, card, or dice games. They should devise and report on experiments to support their conclusions.
- Students investigate assertions such as "Snakes and Ladders is an unfair game". Working in groups, students devise a systematic way to examine the game. They give oral presentations containing an evaluation of such things as the number of snakes vs the number of ladders, the total number of squares gained by going up on ladders vs the total number of squares lost by going down on snakes, and so on.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest ;
- collect appropriate data;
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data;
- collect and display time-series data.

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays;
- evaluate others' interpretations of data displays;
- make statements about implications and possible actions consistent with the results of a statistical investigation.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- estimate the relative frequencies of events and mark them on a scale;
- find all possible outcomes for a sequence of events, using tree diagrams.

Suggested Learning Experiences

Statistical investigations

Students should be:

- discussing environmental or social issues and posing questions for statistical investigation;
- planning manageable and relevant statistical investigations, using strategies such as plan, do, check, and act (Deming cycle);
- talking about samples and sampling, including limitations, and discussing and devising ways of obtaining a representative sample;
- discussing sources of error, including question design and bias, and exploring the influence of question design;
- collecting and presenting discrete data in a variety of ways, for example, pictographs, bar graphs, strip graphs, pie graphs, and presenting displays which compare data, for example, back-to-back stem-and-leaf graphs, and 2 x 2 arrays;
- organising measurement data into classes and presenting it in frequency tables and histograms;
- carrying out simple experiments requiring repeated measurements over time to collect time-series data, and displaying it on appropriate graphs, for example, recording the temperature at 10 a.m. every day, or the height of a pot-grown silver beet plant each day.

Interpreting statistical reports

Students should be:

- considering features of data displays, including the number of clusters and outliers, and identifying the mode; estimating mean and median; using technology where appropriate;
- reporting on statistical investigations, using diagrams, words, and numbers;
- developing an awareness of the ways in which statistical information is used.

Exploring probability

Students should be:

- finding relative frequencies from frequency tables, finding the proportion of the sample in a given category, and recording them on different scales;
- using relative frequencies to make predictions in both words (such as “very likely”, “less likely”) and numbers (such as “nine times out of ten” or 90%);
- displaying and describing sets of possible outcomes, using tree diagrams.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students discuss an issue of interest or concern, for example:
  - Are there trends which would indicate that global warming is taking place?
  - Is their a need for more rubbish or recycling bins to be placed around our school? Students make up questions related to this issue, such as “What types of litter are prevalent in our school?” “Which areas in our school are problems?” “What are some possible solutions?” For example, working in groups students decide how to categorise rubbish, collect and count items of rubbish, summarise their data on an appropriate display, and present their findings to the board of trustees.

  Using this example, teachers could assess students’ ability to:
  - plan a statistical investigation arising from the consideration of an issue or an experiment of interest (S4);
  - collect appropriate data (S4);
  - choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (S4);
  - report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (S4);
  - make statements about implications and possible actions consistent with the results of a statistical investigation (S4);
  - pose questions for mathematical exploration (MP4);
  - interpret information and results in context (MP4).

- Working in groups, students conduct and report on investigations, for example, a sweets investigation. Without opening the packet of sweets, what mathematical questions can one ask about the sweets? Choose questions for further exploration such as “Do all packets have the same number of sweets?” Predict and record possible answers to the questions. Open the packets and record the number of sweets in each packet. Students choose how to display the results. They discuss the original predictions in the light of the actual data.

  Using this example, teachers could assess students’ ability to:
  - choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (S4);
  - report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (S4);
  - interpret information and results in context (MP4).

- Students use a tree diagram to solve a problem. For example, given three types of drink, two types of bread, and four types of spread, how many different breakfasts are possible?

  Using this example, teachers could assess students' ability to:
  - find all possible outcomes for a sequence of events using tree diagrams (S4);
  - use their own language, and the language of mathematics and diagrams, to explain mathematical ideas (MP4).

Sample Development Band Activities

- Students collect as many different graphs, charts, and tables as they can and classify the data presentations on a scale from "clear and helpful" to "unclear and misleading". In cases where the classification is "unclear" they should re-present the data in a clearer or more accurate way, and comment on the way in which their presentation is more informative.
- Students undertake a probability experiment related to an extra-curricular activity in which they are involved. For example, students might present an analysis of their team's goal-scoring record over the season, and draw up probability tables to predict the likelihood of certain players scoring at their next appearance.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- plan and conduct statistical investigations of variables associated with different categories within a data set, or variations of variables over time;
- consider the variables of interest, identify the one(s) to be studied, and select and justify samples for collection;
- find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range;
- discuss discrete and continuous numeric data presented in quality displays;
- collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs.

Suggested Learning Experiences

Statistical investigations

Students should be:

- talking about and formulating statistical questions, including those arising from consideration of graphs and calculations and those that involve comparisons of different, but related, data, and variation over time;
- planning investigations, taking into consideration different sampling methods and appropriate technology, and exploring different sample choices and their implications;
- collecting, classifying, and organising data, including using published data such as year books, meteorological records, medical records, Department of Statistics records, and sports statistics;
- matching appropriate displays to data type;
- presenting data in a variety of appropriate ways, including picture graphs, dot plots, bar graphs, stem-and-leaf graphs, strip graphs, pie graphs, tally charts, frequency tables, and histograms;
- devising ways to display data showing comparisons of variables in related categories and using conventional displays such as composite bar graphs, back-to-back stem-and-leaf graphs, composite dot plots, and paired box-and-whisker graphs;
- devising ways to display data showing variations of variables over time and using conventional time-series displays;
- talking about and considering the features of data displays and selecting the appropriate data measures for calculation;
- analysing and interpreting data (using technology such as spreadsheets and statistics software, where appropriate), finding and using measures of central tendency (mean, median, mode), and spread (quartiles, range), and using their own language to discuss features of the shape (for example, clustered, symmetrical, skewed, and bi-modal);
- preparing and presenting oral and written reports, using computer software, where appropriate.

Statistics

Achievement Objectives

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- use data displays and measures to compare data associated with different categories;
- make statements about time-related variation as a result of a statistical investigation;
- report on possible sources of error and limitations of an investigation.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- determine probabilities of events based on observations of long-run relative frequency;
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck;
- predict the outcome of a simple probability experiment, test it, and explain the results;
- find the probability of a given sequence of events, using tree diagrams.

Suggested Learning Experiences

Interpreting statistical reports

Students should be:

- reflecting on their investigations and seeking ways to develop and strengthen them;
- making statements about and exploring the ways in which two data sets are different and similar, from consideration of data displays that compare them;
- considering investigations and data displays that show changes in populations over time, identifying the key features that show the change, talking about the change with others, and making statements about any implications of the change;
- generalising the sample results to a larger population and discussing the reasonableness of the generalisation;
- reflecting on the results of investigations, posing new questions and identifying implications and suggestions for action;
- evaluating raw data to determine whether adjustments are necessary, for example, actual dollar values versus inflation-adjusted values in comparing prices over time, or actual rates versus per capita rates;
- investigating ways in which statistical information is presented by the media and other sources, and recognising and identifying sources of deception in misleading graphs and their accompanying statements.

Exploring probability

Students should be:

- developing a concept of probability, using computer software where appropriate, by finding probabilities of everyday events based on past results;
- designing and conducting experiments to determine probability as long-run frequency;
- discussing situations that give rise to predictable probabilities and estimating the probabilities, including consideration of some simple games of chance involving dice, coins, or cards, and using tree diagrams to explore situations involving two or more events;
- investigating probability in social contexts.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students organise, analyse, summarise, interpret, and display data from an attitude survey. For example, they might investigate their own attitudes and opinions about parenting by completing starter statements such as "The hardest thing about being a parent is . . . ." and "The thing I value most about my parent(s) is . . . ." Students complete each starter and record their responses on a slip of paper. Students devise a scheme for categorising responses then collect, categorise, interpret, and display the responses.

  Using this example, teachers could assess students' ability to:
  - consider the variables of interest and identify the one(s) to be studied, and select and justify samples for collection (S5);
  - use data displays and measures to compare data associated with different categories (S5);
  - report on possible sources of error and limitations of an investigation (S5);
  - pose questions for mathematical exploration (MP5);
  - interpret information and results in context (MP5).

- Students explain why a game is unfair and suggest ways it could be made fair. For example, a coin is tossed. If the outcome is heads, player A wins. If the outcome is tails, the coin is tossed again. If the second toss is heads, player A wins; if tails, player B wins. Show this situation, using a tree diagram, and calculate the probability of A winning.

  Using this example, teachers could assess students' ability to:
  - determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (S5);
  - find the probability of a given sequence of events, using tree diagrams (S5).

- Students carry out a simple probability experiment. For example, they determine the probability of a tossed coin landing within a square on a square grid. Students predict the probability, decide on the number of trials, carry out the experiment, record and graph results, and then present their conclusions, including a justification of the number of trials used.

  Using this example, teachers could assess students' ability to:
  - determine probabilities of events based on observations of long-run relative frequency (S5);
  - choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (S4).

- Students analyse data and present data displays from two or more populations of their own choice, such as the comparative achievement in mathematics by males and females, or the length of fibres from the fleeces of two breeds of sheep.

  Using this example, teachers could assess students' ability to:
  - plan and conduct statistical investigations of variables associated with different categories within a data set, or variations of variables over time (S5);
  - collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs (S5);
  - use distinctive features of data displays and measures to compare data associated with different data (S5);
  - pose questions for mathematical exploration (MP5);
  - make conjectures in a mathematical context (MP5).

- Students examine and compare categories within a data set. For example, having classified and counted all the students at their form level by their eye colour, they choose an appropriate method to display the comparison between the numbers of blue-eyed students and brown-eyed students in each class.

  Using this example, teachers could assess students' ability to:
  - find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range (S5);
  - collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs (S5);
  - use data displays and measures to compare data associated with different categories (S5);
  - interpret information and results in context (MP5);
  - record in ways that are helpful for drawing conclusions and making generalisations (MP5).

Sample Development Band Activities

- Students analyse games of chance to determine whether they are fair or whether, for example, they are loaded in favour of the "banker". They could invent games based on tossing coins or rolling dice and determine and explain each player's probability of winning, or their expected winnings in the long term.
- Students investigate random walks.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- formulate statistical questions about situations involving possible relationships between variables;
- formulate questions about variations over time in continuous processes;
- identify data collection methodology;
- collect bi-variate measurement and discrete number data, and clearly and concisely communicate the significant features in appropriate displays, including scatter plots.

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- make and justify statements about relationships between variables in a sample as a result of a statistical investigation;
- identify long-term and short-term features in time-series data;
- suggest improvements in the investigation, design, data collection, or display, where possible inferences are inconclusive.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- use tables of multi-variate data from social contexts to find the probabilities of everyday events or the proportion of outcomes in a given category;
- determine the theoretical probabilities of the outcomes of both exclusive and independent events such as the rolling of a die followed by the drawing of a card from a deck;
- use probability trees to calculate conditional probabilities.

Suggested Learning Experiences

Statistical investigations

Students should be:

- constructing reports involving pictures, words, and numbers;
- constructing quality graphics for communication;
- collecting multi-variate data sets (for example, name, age, gender, height, mass), studying pairs of variables, and presenting bi-variate measurement and category data in appropriate ways including scatter plots and arrays;
- choosing, designing, undertaking, and evaluating statistical investigations. Suitable investigations could involve predictions from trend lines in time-sequence data, extractions from tables of multi-variate data, designing questionnaires, and using spreadsheets and statistical software.

Interpreting statistical reports

Students should be:

- exploring and discussing the strengths and weaknesses of statistical investigations, including the success of displays in communicating the features of the data, the clarity of reports, and any sources of error and bias, and suggesting improvements;
- considering data displays that show relationships between variables, including scatter plots, making statements about the relationship, and making predictions;
- evaluating statistics in the news media, and in technical and financial reports.

Exploring probability

Students should be:

- developing a concept of probability by exploring games and activities which involve chance, using computers where appropriate;
- finding and interpreting probabilities, using random experiments, tree diagrams, arrays, and tables of data drawn from practical contexts;
- exploring the concept of conditional probability by practical experiment and by using tree diagrams, including situations with and without replacement.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students conduct and report orally and in writing on an investigation. For example:
  - They determine the future needs of their school. This could include predicting numbers of enrolments for the next decade by investigating enrolment numbers and retention rates at each level of the contributing schools and kindergartens.
  - They investigate the relationship between the amount of practice and the performance of a simple task, such as a hand-eye co-ordination task. They extend this to more difficult tasks and make comparisons.
  - There is a proposal to manufacture fingerless gloves. Students undertake market research to find out how many of each size to manufacture.

  Using this example, teachers could assess students' ability to:
  - formulate statistical questions about situations involving possible relationships between variables (S6);
  - plan and conduct statistical investigations of variables associated with different categories within a data set, or variations of variables over time (S5);
  - record information in ways that are helpful for drawing conclusions and making generalisations (MP6).

- Students draw and interpret scatter plots derived from familiar contexts, for example, the heights and weights of 30 students in their 11th year at school. They estimate the weight of an unknown student, given the height, and discuss the reasons for their inference.

  Using this example, teachers could assess students' ability to:
  - collect bi-variate measurement and discrete number data, and clearly and concisely communicate the significant features in appropriate displays, including scatter plots (S6);
  - make and justify statements about relationships between variables in a sample as a result of a statistical investigation (S6);
  - make conjectures in a mathematical context (MP6).

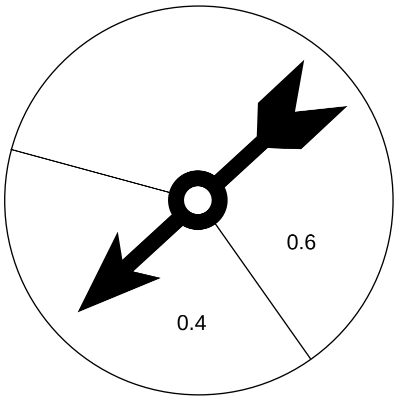
- Working in groups of three, students collect data on a topic, and use their data to support different sides in a debate. For example, city students could collect data showing the arrival and departure times of planes over a 24-hour period at a nearby airport. One side might argue a case for a more stringent airport curfew, while the other argues for a more relaxed curfew. Both sides would use statistical arguments to support their case while looking for errors or invalid inferences in the other case.

Using this example, teachers could assess students' ability to:
- formulate statistical questions about situations involving possible relationships between variables (S6);
- make and justify statements about relationships between variables in a sample as a result of a statistical investigation (S6);
- identify long-term and short-term features in time-series data (S6);
- pose questions for mathematical exploration (MP6);
- prove or refute mathematical conjectures (MP6);
- report the results of mathematical explorations concisely and coherently (MP6).

• Students present statistical information collected from published data. For example, they collect a table of multi-variate social data that interests them. They pose questions, suggest implications and possible action, analyse the data to test their questions, and then write a letter suitable for publication in a newspaper, explaining and justifying their conclusions.

  Using this example, teachers could assess students' ability to:
  - use tables of multi-variate data from social contexts to find the probabilities of everyday events or the proportion of outcomes in a given category (S6);
  - make and justify statements about relationships between variables in a sample as a result of a statistical investigation (S6);
  - interpret information and results in context (MP6).

• Students calculate probabilities (a) using a spinner to simulate the situation, and (b) using a tree diagram. For example, the results of many games of tennis between 2 players suggest that player A has a 0.6 chance of winning a set against player B. The first player to win 2 sets, wins. Find the probability that player A wins the match.



  Using this example, teachers could assess students' ability to:
  - determine the theoretical probabilities of the outcomes of both exclusive and independent events such as the rolling of a die followed by the drawing of a card from a deck (S6);
  - use probability trees to calculate conditional probabilities (S6);
  - solve practical problems involving decimals and percentages (N5).

Sample Development Band Activities

- Students use a simulation to model the inoculation of a population and investigate what is needed to ensure success for the programme.
- Students investigate the role of statistics in describing and shaping aspects of public life, for example, forecasting economic trends, forecasting election results, describing or influencing consumer taste, developing public perception, and developing policy on controversial issues.
- Students investigate the derivation and use of standard measures of correlation for testing the strength of the relationship in sets of bi-variate data. For example:
  - They might use Spearman's formula for rank correlation to determine whether male and female form 5 students rank their subject preferences similarly.
  - They might collect data to compare armspan with height, hair length with height, hair length with armspan. They graph the data on scatterplots and discuss what these show about the relationships between the pairs of variables, then calculate Pearson's correlation co-efficient for each set of data, and discuss how these correspond to the relationships shown by the graphs.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- plan a statistical investigation to make inferences about a population or experimental situation;
- design and justify sample selection and data collection methods;
- collect data, present it visually, and discuss prominent features of the data;
- calculate sample statistics, including mean and standard deviation, and verify these by reference to a data distribution;
- highlight features in time-series graphs by simple algebraic transformations.

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- analyse and discuss statistically-based inferences about populations or experiments;
- identify causes of long-term and short-term trends in time-series data extracted from reference sources or experiments.

Exploring probability

Within a range of meaningful contexts, students should be able to:

- simulate situations using dice or random number generators to calculate probabilities of outcomes;
- recognise situations where the normal distribution is a suitable mathematical model and use this model to solve problems;
- reduce a normal distribution to standard normal form and use tables of normal probabilities.

Suggested Learning Experiences

Statistical investigations

Students should be:

- designing, undertaking, and evaluating statistical investigations involving random sampling, systematic sampling, and non-representative sampling; considering bias; and using different methods of data collection;
- justifying choice of samples to represent populations;
- using technology, such as spreadsheets and graphing software, where appropriate, to investigate and display data;
- analysing their findings, using sample statistics, including the standard deviation, and reporting their findings;
- investigating the features of time-series graphs by moving averages, for example, dividing one series by another.

Interpreting statistical reports

Students should be:

- investigating graphs of time-series data, identifying inherent (for example, seasonal) and assignable (abrupt changes or drift) variability, and discussing possible causes for assignable variability;
- evaluating statistics presented in the news media, and in technical and financial reports, and confidently expressing reasoned opinions on them.

Exploring probability

Students should be:

- investigating the generation and properties of random numbers;
- investigating and solving problems about a variety of populations that can be modelled by the normal distribution;
- using tree diagrams and tables to model situations where outcomes are not necessarily equally likely;
- simulating situations which can be modelled by tossing coins or throwing dice.

Statistics

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students carry out a practical survey and make a presentation of their findings. For example:

  - They might investigate the local community's need for a skate board ramp. They could survey the use of skate boards, design and cost a ramp, and make a presentation to the local council.

  - They investigate learning in mathematics by interviewing younger people about the strategies they use when solving mathematical tasks, and by reflecting on their own approaches to topics in mathematics. They explore the level of retention in each case, trial the effects of alternative approaches to the learning of mathematics, and present their conclusions to the class.

  Using this example, teachers could assess students' ability to:
  - plan a statistical investigation to make inferences about a population or experimental situation (S7);
  - collect data, present it visually, and discuss prominent features of the data (S7);
  - effectively plan mathematical exploration (MP7);
  - record information in ways that are helpful for drawing conclusions and making generalisations (MP7).

- Having gathered data on a situation of interest, the students use calculators, and/or spreadsheets or other appropriate software, to analyse the data, calculate appropriate sample statistics, and draw graphics. They present their conclusions to the class visually and orally.

  Using this example, teachers could assess students' ability to:
  - calculate sample statistics including mean and standard deviation and verify these by reference to a data distribution (S7);
  - use equipment appropriately when exploring mathematical ideas (MP7).

- Students critically evaluate time-series data presented in media reports or references such as The New Zealand Year Book, and discuss the statistical significance of trends which may be evident. They present reasoned opinions on the causes and likely long-term impact of variations and trends.

  Using this example, teachers could assess students' ability to:
  - identify long-term and short-term trends in time-series data (S6);
  - identify causes of long-term and short-term trends in time-series data extracted from reference sources or experiments (S7);
  - interpret information and results in context (MP7);
  - report the results of mathematical investigations concisely and coherently (MP7).

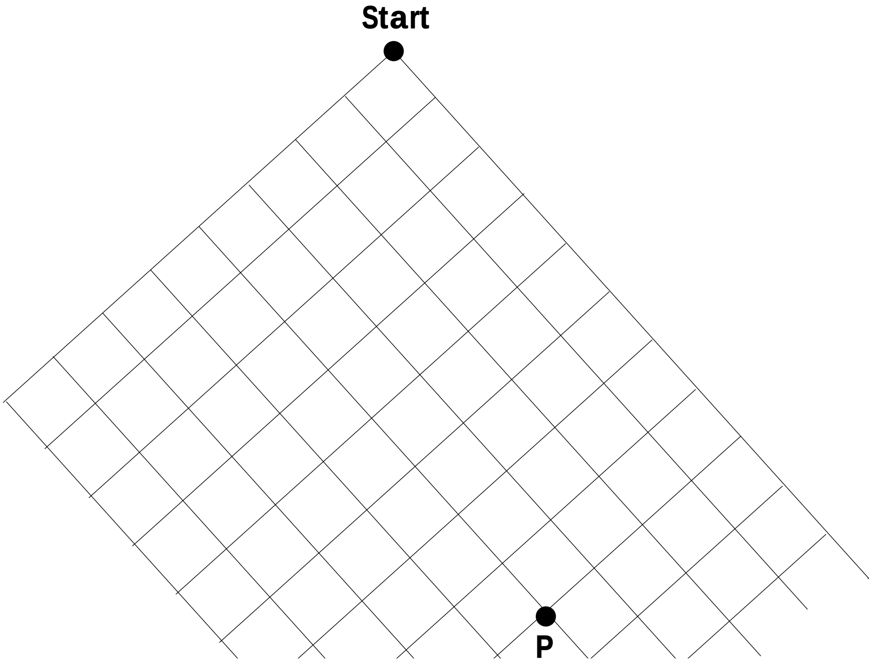
- Students simulate a situation by rolling dice, using random number tables, spreadsheets, or other methods. For example, students use simulation to find how many cereal packets they should buy in order to get a complete set of six cards.

  Using this example, teachers could assess students' ability to:
  - simulate situations, using dice or random number generators to calculate probabilities of outcomes (S7);
  - use equipment appropriately when exploring mathematical ideas (MP7).

- Students model a situation for which they have a large sample of data by a normal distribution. For example, a sample of 500 concrete trucks, each supposedly delivering 3 cubic metres of concrete, has been weighed. The net mass of concrete in the trucks appears to be normally distributed about a mean of 2 700 kg with a standard deviation of 150 kg. Students apply the properties of a normal distribution to calculate and apply probabilities that the mass of concrete in a truck measured at random will or will not lie within or beyond certain limits, or, for example, the probability that three trucks in a row would deliver too little concrete for a job.

  Using this example, teachers could assess students' ability to:
  - recognise situations where the normal distribution is a suitable mathematical model and use this model to solve problems (S7);
  - reduce a normal distribution to standard normal form and use tables of normal probabilities (S7).

- Students model a physical situation and draw conclusions. For example, ants move down a wire grid as illustrated in the following diagram. Students investigate the number of routes to the node P. They investigate the problem by conducting random walks (tossing a coin at each node), then solve it by solving a simpler problem first and looking for a pattern.



Using this example, teachers could assess students' ability to:
- simulate situations, using dice or random number generators to calculate probabilities of outcomes (S7);
- effectively plan mathematical exploration (MP7);
- devise and use problem-solving strategies to explore situations mathematically (MP7);
- make conjectures in a mathematical context (MP7);
- prove or refute mathematical conjectures (MP7).

Sample Development Band Activities

- Students investigate the use of statistical techniques in, for example, market research and life insurance.
- Students investigate questionnaire design. They develop and trial interview schedules to see if they serve their intended purpose. They investigate how different forms of questions affect responses.
- Students develop probability distributions for discrete random variables, such as the product obtained when 2 dice are rolled.

Statistics

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- plan a statistical investigation to estimate (2σ) confidence intervals (margins of error) for the estimation of population parameters;
- report on a statistical experiment to investigate the consistency of samples in a process which is continuous over time.

Interpreting statistical reports

Within a range of meaningful contexts, students should be able to:

- explain differences between sample means and the population mean;
- recognise and explain sources of bias and unreliability in sampling situations;
- evaluate and explain the meaning of confidence intervals in estimating population parameters and in using samples for quality control;
- identify situations where the sample may not have come from a clearly defined population, and discuss the relevance of the findings.

Suggested Learning Experiences

Statistical investigations

Students should be:

- discussing the sensible use of location, spread, and visual displays of data in a wide variety of applications, and interpreting their use in the news media, and in technical, business, and their own reports;
- performing practical experiments to verify sampling distributions and discussing the central limit theorem;
- using sample statistics:
  - as estimates of population parameters;
  - to calculate standard error and apply to confidence intervals using the 2-standard error rule;
  - to investigate large sample confidence intervals for the population mean, population proportion, and the difference between the means of two populations;
  - to consider bias and sampling error;
- simulating a continuous process, for example, by cutting pieces of string to a specified length, then analysing samples to determine the consistency of the process;
- investigating industrial applications of sampling to quality and/or process control.

Interpreting statistical reports

Students should be:

- investigating "margins of error" in survey reports and opinion polls;
- critically analysing data and reports to determine the point of view of the author, and re-presenting data for different target audiences;
- using appropriate technology to interpret data which does not assume any particular distribution.

Statistics

Achievement Objectives

Exploring probability

Within a range of meaningful contexts, students should be able to:

- solve problems, using techniques which simulate a probability situation;
- define and use the basic terms and concepts of probability;
- choose the appropriate distribution (binomial, Poisson, or normal) to model a given situation, calculate probabilities and expected values, and make predictions using the model;
- calculate and interpret expected values for practical situations.

Suggested Learning Experiences

Exploring probability

Students should be:

- using simulations to model real or imaginary situations;
- exploring the basic concepts of probability (trial, outcome, sample space, event) including disjoint (exclusive) events, complementary events, and exhaustive events, and should be calculating probabilities of combined events, conditional probabilities, and independent events;
- exploring discrete random variables and their use in, for example, life insurance, games of chance, events involving risk;
- formalising the expected value and variance of a linear function of a random variable and of sums of independent random variables;
- discussing, and experimenting with different probability distributions (rectangular, binomial, Poisson, normal), exploring their applications, developing criteria for the application of each and the relationships between them, and using tables of probabilities for these distributions;
- creating probability distributions for given variables and using them to solve problems;
- exploring the use of probability models in simulations. For example, they could investigate the use of a spreadsheet to apply the Poisson distribution in a simulation of queuing at a supermarket checkout during various time periods in a day.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students investigate the application of confidence limits in Quality Management. For example, they simulate a canning operation by filling a large number of identical containers (for example, paper cups) with sand. The net mass of sand in each can is measured and the data analysed to establish parameters for the mass of sand in a can. Using this data, they produce average and range charts for the process of canning sand. To carry out the simulation they could repeatedly fill samples of cans and measure their statistics, comparing variations with the charts. They pose questions about the process. For example, are there noticeable variations when new "fillers" are employed?

  Using this example, teachers could assess students' ability to:
  - plan a statistical investigation to estimate (2σ) confidence intervals (margins of error) for the estimation of population parameters (S8);
  - report on a statistical experiment to investigate the consistency of samples in a process which is continuous over time (S8);
  - evaluate and explain the meaning of confidence intervals in estimating population parameters and in using samples for quality control (S8);
  - identify causes of long-term and short-term trends in time-series data extracted from reference sources or experiments (S7).

- Students devise and test a mathematical model of a practical problem. For example, it is claimed that if all cars at a traffic light stopped 2 metres apart and moved simultaneously at the same speed when the lights turned green, then 4 times as many cars would get through the lights on each phase. They investigate this, using methods such as simulation and on-site analysis, and report with recommendations.

  Using this example, teachers could assess students' ability to:
  - solve problems, using techniques which simulate a probability situation (S8);
  - choose the appropriate distribution (binomial, Poisson, or normal) to model a given situation, calculate probabilities and expected values, and make predictions using the model (S8).

- Students decide which distribution is the best model for a given situation and use an appropriate strategy to solve a problem. For example, a pest-eradication expert claims she can remove cockroaches from any house 80% of the time. The health inspector decides to test this claim by monitoring 120 houses, and 79 houses are found to be free of cockroaches. Students decide which distribution is the best model, giving reasons, and test the expert's claim.

  Using this example, teachers could assess students' ability to:
  - choose the appropriate distribution (binomial, Poisson, or normal) to model a given situation, calculate probabilities and expected values, and make predictions using the model (S8);
  - evaluate and explain the meaning of confidence intervals in estimating population parameters and in using samples for quality control (S8).

Sample Development Band Activities

- Students develop theoretical probability distributions. For example, they use simulation and model building to develop the binomial distribution.
- Students investigate the use of other probability distributions such as Student's t-distribution.
- Students investigate probability in a number of complex situations. For example:
  - As part of a recycling campaign, they are to design a fruit machine that will accept aluminium cans instead of money and that will give prizes. Design the machine on the assumption that it is to be non-profit making and that each can is worth \$0.01. What changes would need to be made if the machine is to make a 20% profit and cans are worth \$0.015?
  - Students find the probability that at least 2 people in a group have the same birthday.
  - Students investigate situations where the normal distribution, with an appropriate continuity correction, can be used as an approximation for situations more precisely modelled by the Poisson distribution or the binomial distribution.

Glossary of terms

Students are not expected to know all of these terms. While many are used loosely in everyday situations, teachers should know their mathematical meanings.

Absolute value The value of a number when the sign is not considered.

Acute angle An angle whose size is between 0° and 90°.

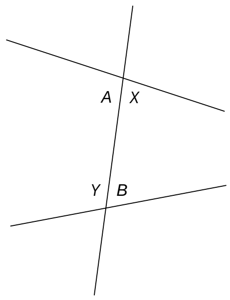
Acute-angled triangle A triangle with three acute angles.

Addend Any one of a set of numbers to be added.

Adjacent angles Two angles which do not overlap, displayed by 3 rays with the same end point.

Algorithm A standard procedure for performing a task or solving a problem.

Alternate angles If two lines are cut by another line, then the inside angles on opposite sides of the other line are called alternate angles.



Thus A and B or X and Y are alternate angles.

Altitude of a triangle The perpendicular distance from a vertex to the opposite side of a triangle is called the altitude (or height). Sometimes the line segment is referred to as the altitude.

Analogue clock A traditional clock with minute and hour hands.

Angle The union of two rays with a common end point (called the vertex). The size (or measure) depends on the amount of rotation from one ray to the other — this amount is also sometimes referred to as the angle.

Apex The highest point of a figure with respect to a chosen base.

Arc Part of a curve.

Area The size (or measure) of a surface expressed as a number of square units.

Arrow diagram (or graph) A diagram or graph using arrows to show a relation.

Assignable causes (of variability) Causes of variability in data collected over time which could not have been predicted ahead of time. Usually associated with an abrupt change or drift in a process.

Average of a set of numbers The sum of all the numbers divided by the number of numbers. The term average is also used as a general term for mean, median, and mode, that is, for a single number which represents a collection of numbers.

Axiom a self-consistent, self-evident statement that is universally accepted as true (hence, *axiomatic*).

Axis (axes) The line(s) which form the framework for a graph.

Axis of symmetry A line which divides something in half so one half is the mirror image of the other half.

Back-to-back stem-and-leaf graphs Made by using the same stem for two sets of data. For example:

	3	2	0	5				
		9	8	5	9			
4	1	0	0	6	2	4		
	7	6	6	6	7	8	8	
			2	7	1	1	4	4
		9	6	7	8			
				8	2	4		
				8	9			
Pulse Rate before exercise				9	2	Pulse Rate after exercise		

Bar graph A graph which uses horizontal or vertical bars to represent information. (The bars are of equal widths with lengths proportional to frequencies or probabilities. If the bars are vertical it may be called a column graph.)

Base The base of a figure (or solid) is the side (or face) on which the shape is imagined to stand.

Base (number) The number on which a place-value system is built. The base of our number system is ten.

Basic Facts The addition facts up to 9 + 9 and the multiplication facts up to 10 x 10.

Bearing Direction, clockwise from north, named by an angle in degrees.

Billion One thousand million (10^9).

Binary system A base-two number system.

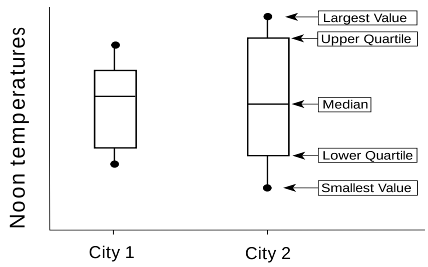
Bisect To cut into two congruent parts.

Bisector A line (or ray) that cuts a figure or line segment or angle into two congruent parts.

Bivariate data Data which is classified according to two variables. For example, the heights and weights may be found for each student.

Block graph A bar or column graph constructed from blocks.

Box-and-whisker graph A graph which may be used to give an immediate impression of the spread of data when comparing populations. The lower and upper quartiles are the numbers one quarter and three quarters of the way along, respectively, when the values are listed in order.



Braces The symbol { } used to stand for the word set. The elements of a set are written inside the braces. For example, {1, 2, 3, 4, ...} is the set of counting numbers.

Brackets The symbols ( ) [ ] { } used for grouping things together.

Capacity The interior volume of a container.

Cardinal number The number that tells how many objects are in a set.

Category data Data which can be organised into distinct categories, for example, foods could be organised into categories such as meat, fish, vegetables, fruit, cereal.

Central tendency A general name given to different ways of describing the middle values of data.

Chord A line segment with both end points on a circle (or curve).

Circle A plane figure bounded by a set of points equidistant from a fixed point (the centre). Circle is also used to refer to the boundary only.

Circumference The boundary of a circle. Circumference is also used for the length of this boundary.

Clinometer An instrument for measuring vertical angles.

Clockwise The description of rotation by comparing with the hands of a clock. (The opposite is either anti- or counter-clockwise.)

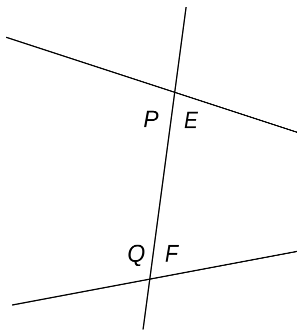
Closed curve A curve that can be traced by beginning and ending at the same point.

Closed sentence A sentence that is either true or false.

Closed surface A surface that encloses a part of space (called the interior).

Co-efficient The numerical co-efficient of a term is the number part of it, for example, $^-3x^2$ has co-efficient $^-3$.

Co-interior angles If two lines are cut by another then the inside angles on the same side of the other line are called co-interior angles.



Thus *P* and *Q* or *E* and *F* are co-interior angles.

Collinear points Collinear points are points on the same straight line.

Column graph A bar graph with vertical bars.

Common denominator For two or more fractions a common denominator is a number into which all the denominators divide exactly.

Common factor For two or more numbers a common factor is a number which divides into all the numbers exactly.

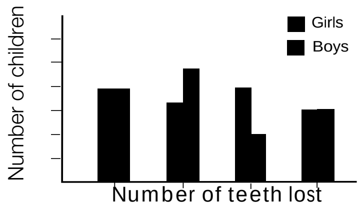
Compact numeral The usual short form of a numeral using place values.

Compass An instrument which shows direction.

Compasses (pair of) An instrument used to draw circles and to mark off equal lengths (often called a compass for short).

Complementary angles Two angles whose measures add to 90°. One is called the complement of the other.

Composite bar graph A bar graph for comparing two or more sets of data. For example:



Composite number A whole number greater than 1 that is not prime. (A counting number with 3 or more factors.)

Concentric circles Circles in the same plane with the same centre are concentric.

Concurrent lines Concurrent lines are lines that all pass through one point.

Conditional probability The probability of an event assuming that another has occurred. For example, the probability that a student chosen at random from the school has brown eyes given that he or she has fair hair.

Cone A solid which has a circular base and tapers uniformly to a point called the vertex.

Congruent Figures are congruent if they have the same shape and size. The symbol ≅ means "is congruent to".

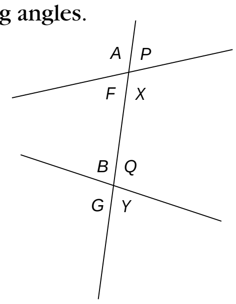
Continuous Data Data which could, in principle, assume any other value between any two given values. It is usually data collected by measurements, such as length. (Compare with discrete data.)

Contradiction Disagreement with a statement by making another statement that cannot be true at the same time, for example, "The moon is square" contradicts "The moon is round".

Converse A statement expressed in the opposite way, for example, "If *A* then *B*" becomes "If *B* then *A*".

Co-ordinates Numbers written in order to give position on a line, a plane, or in space. When two or more, they are usually written in brackets, for example, (5,3) or (2,5,4).

Corresponding angles If two lines are cut by another, then the angles in "corresponding" positions are called corresponding angles.



Thus *A* and *B* or *F* and *G* or *P* and *Q* or X and Y are corresponding angles.

Cube A solid with six congruent square faces. (Sometimes the word is used for the surface only.)

Cube of a number The product when the number is used as a factor three times, for example, 4 cubed is 4 x 4 x 4, or 4^3, or 64.

Cuboid A rectangular prism.

Curve A set of connected points that form a path.

Cylinder A prism of circular cross section.

Data A set of facts, numbers, or information.

Decagon A polygon with ten sides.

Decimal A decimal fraction.

Decimal fraction A numeral that uses place value and a decimal point to name a fraction.

Decimal system A base-ten number system.

Decomposition A method of subtraction where the larger number is rewritten to enable the smaller one to be subtracted.

		61
74	60 + 14	74
—28	— (20 + 8)	—28

Degree The standard unit for measuring angles ($1^o = \frac{1}{36}$ of a complete revolution).

Denominator The bottom line of a fraction (9 is the denominator of $\frac{2}{9}$) .

Density The mass per unit of volume of a material.

Diagonal A line segment that joins two vertices of a polygon, but is not a side.

Diameter of a circle A diameter of a circle is a chord that passes through the centre. (Diameter is sometimes used to refer to the length of the diameter.)

Difference The amount by which one number is greater than another. The difference is found by subtraction.

Digit A symbol used to write numerals.

Digital clock A clock that displays the time in numbers.

Dilation An enlargement or reduction of a shape in which all the linear measures are multiplied by the same number (called the scale factor).

Discount A reduction from the regular price.

Discount rate A discount rate of 20% means the regular price is reduced by 20 per cent of itself.

Discrete data Data which can take only certain values, such as the number of legs on animals. (Compare with continuous data.)

Disjoint sets Sets are disjoint if their intersection is empty, that is, they have no common members.

Distance from a point to a line The length of the perpendicular line segment from the point to the line.

Distortion A transformation which changes the shape, rather than only the size, of a figure.

Dividend If the one number is being divided by a second, then the first number is the dividend, for example, in 12 ÷ 3 = 4, 12 is the dividend.

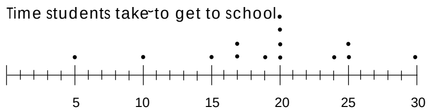
Divisible One number is divisible by another if the first is a multiple of the second.

Divisor If one number is being divided by a second, then the second number is the divisor, for example, in 12 ÷ 3 = 4, 3 is the divisor.

Dodecahedron A polyhedron with twelve faces.

Domain See Mapping.

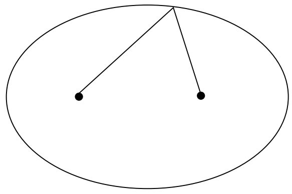
Dot plot Represents outcomes as dots on a scale. For example:



Edge The intersection of two planes.

Element (member) of a set An item in a set.

Ellipse A figure bounded by a curve in which the sum of the distances from any point on the curve to two particular points is always the same.



Empty set The set that has no members. (Its cardinal number is 0.) It is written as ϕ or { }.

Equation An open number sentence stating that two quantities are equal.

Equilaterial triangle A triangle in which all three sides are congruent.

Equivalent fractions Fractions that name the same fractional number.

Equivalent sentences (equations) Sentences (equations) with the same solution set.

Equivalent sets Sets that contain the same number of members.

Estimation An approximate calculation or a judgement of what a result will be.

Euler's relation $V + F - E = 2$ Where V is the number of vertices (or nodes), F the number of faces (or regions), and E the number of edges (or arcs) on a polyhedron (or network).

Expanded numeral A numeral written as the sum of the total value of its digits, for example, 800 + 40 + 2.

Exponent A numeral written above and to the right of another numeral to show how many times the base numeral is used as a factor, for example, in 5^3, 3 is the exponent, $5^3 = 5 \times 5 \times 5 = 125$. Another word for exponent is index.

Exponentiation The act of expressing a number as a base number with an exponent.

Exterior angle An angle formed between a side of a figure and the extension outside the figure of an adjacent side.

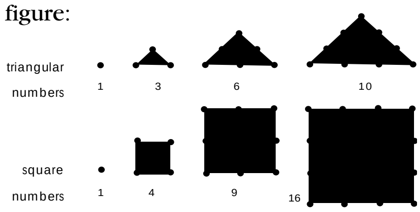
Extrapolation An approximate calculation from known data of others which lie outside the range of the data.

Face A plane surface of a polyhedron.

Face value The value of the number that a digit names. For example, in 57, 5 has face value five.

Factor A number that is multiplied by another number to name a product, for example, 3 x 2 = 6 so 3 and 2 are factors of 6.

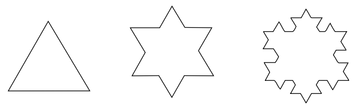
Figurate numbers Numbers that can be represented by dots arranged in a figure:



Finite set A set whose members can all be listed or counted.

Flow-chart A set of instructions in chart form.

Fractal A shape formed by an iterative process giving an infinite perimeter for a finite area. For example:



Fraction A numeral naming a fractional number, usually written in the form $\frac{2}{5}$.

Fractional number A number used to compare a subset and its set or a part to the whole.

Frequency The number of times an item appears.

Frequency table A table showing the frequencies of data items in categories or intervals.

Function A set of ordered pairs where each first element of the pairs occurs once only, for example, {(0,2),(1,3),(2,4)...}.

Geodesic Built of short struts along the shortest possible lines on a surface.

Golden section The ratio $\frac{a}{b}$ when $\frac{a}{b} = \frac{(a+b}{b}$

Graph A picture or diagram which shows information.

Greatest common factor The greatest number in the set of common factors of two or more numbers.

Great circle A circle formed by the intersection of a sphere with a plane through its centre.

Grid A frame of spaced parallel lines (or curves), usually in two directions, to form a reference basis for maps and graphs.

Hectare A unit of area. One hectare is the area of a square measuring 100 m by 100 m (1 ha = 10 000 m^3).

Heptagon A polygon with seven sides.

Hexagon A polygon with six sides.

Highest common factor (HCF) Another name for greatest common factor.

Histogram A bar chart of continuous data for which the area of the bars is proportional to the frequency of the data category represented.

Horizontal line A line perpendicular to a vertical line.

Hypotenuse The longest side of a right-angled triangle, that is, the side opposite the right-angle.

Icosahedron A polyhedron with twenty faces.

Index Another name for an exponent.

Inequation A number sentence stating that one number is greater (or less) than another. (In fact, the numbers may be related by $<$, ∂ , $<$, $>$, $\geqslant$, ∞, $\nless$, $\nleqslant$, $\ngtr$, $\ngeqslant$)

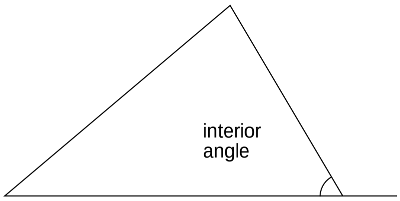
Infinite set A non-empty set that is not finite.

Inherent variability Variability in a time-series data set which is expected because of the nature of the measurement, for example, average weekly temperatures have inherent seasonal variability.

Integer A number of the set {...,$^-3$, $^-2$, $^-1$, 0, 1, 2, 3,...}.

Integer pairs An ordered pair in which the numbers are integers, for example, (2, $^-3$).

Interior angle An angle inside a figure.



Interpolation An approximate calculation, from known data, of other values which lie inside the range of the data.

Intersect Two or more figures are said to intersect (or cut) if they have at least one point in common.

Intersection The intersection of sets A and B, written $A \cap B$, is the set of elements in both A and B.

Invariant Remains unaltered.

Inverse element The inverse element is the element that has the reverse effect for a given binary operation, for example, in addition the inverse of $^+2$ is $^-2$.

Inverse operation The operation which reverses the original operation, for example, the inverse of plus is minus.

Isomorphism A change or re-arrangement of a set or mathematical structure which leaves the basic structure the same.

Isosceles triangle A triangle with two congruent sides.

Iterative Characterised by frequent repetition (of the process).

Kilo A prefix meaning 1000.

Kite A quadrilateral that has two pairs of congruent adjacent sides.

Latitude Represented by circles on a sphere, parallel to the equator, which can be used to refer to distance from the equator.

Least common multiple (LCM) The least counting number in the set of common multiples of two or more numbers.

Line An undefined term. A straight curve that goes on indefinitely in both directions. If it goes through A and B it is written $\overleftrightarrow{AB}$.

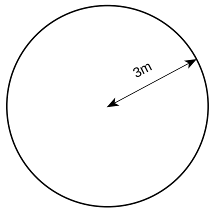
Line graph A graph formed by line segments which join the points representing data.

Line segment A part of a line that has two end points and includes all the points between these points. If the line segment has end points A and B it is written $\overline{AB}$.

Line symmetry A shape has line symmetry if it can be reflected into itself.

Linear equation An equation that has a straight-line graph.

Locus A set of points mapping out a path. For example, the locus of points 3 m from a given point is a circle.



Longitude Represented by circles on a sphere with the polar axis as diameter, which can be used to refer to distance east or west of Greenwich.

Lowest common multiple (LCM) Another name for least common multiple.

Maori kupu

haka	*dance*
hāngi	*earth oven*
hui	*gathering*
kaimoana	*sea food*
kaumátua	*elder*
kina	*sea eggs*
kowhaiwhai	*rafter patterns*
kūmara	*sweet potato*
mangó	*shark*
marae	*meeting ground*
pātiki	*flounder*
raranga	*weaving*
tāniko	*embroidered border*
tipare	*headband*
tukutuku	*panel designs*
waiata	*song*
whai	*string games*
whakairo	*carving*
wharenui	*big meeting house*

Mass The mass of an object is a measure of the amount of matter in it. (The term weight is often used, imprecisely, for mass).

Mean The mean of a set of scores is the sum of the scores divided by the number of scores. (The term average is sometimes used instead of mean.)

Median The number which comes in the middle of a set of numbers when they are arranged in order.

Median of a triangle A line drawn from a vertex of a triangle to the mid-point of the opposite side.

Mediator Another name for perpendicular bisector.

Member of a set A member (or element) of a set is an item in a set.

Metric system See Systéme Internationale d'Unites.

Mid-point of a line segment The point on a line segment that is the same distance from both end points.

Minute (angles) One minute (written 1') equals $\frac{1}{60}^{o}$.

Mirror line See axis of symmetry.

Mixed number A number which is part whole and part fraction.

Mode The number that appears most often in a set of data.

Multi-layer time-sequence graphs Two time-sequence curves plotted on the same graph for the purpose of comparing the two situations.

Multiple A multiple of a number is a product of that number and a whole number (or natural number, according to the text being used).

Multivariate data Data which is classified according to more than two variables. For example, the heights, weights, and gender may be found for each student.

Natural number One of the counting numbers, {1,2,3,...}.

Net A pattern of polygons that can be folded to form (the surface of) a polyhedron.

Nomograph (or Nomogram) A device to find a third variable when given two others involved in a known relation, usually by placing a ruler across scales on a chart or graph.

Number line A line with points labelled by numbers.

Number plane A plane with points labelled by ordered number pairs.

Numeral A name or symbol used for a number.

Numerator The top line of a fraction (2 is the numerator of $\frac{2}{9}$).

Obtuse angle An angle with size between 90^{o} and 180^{o}.

Octagon A polygon with eight sides.

Octahedron A polyhedron with eight faces.

Odds A gambling term for a comparison of probabilities.

Open number sentence A number sentence that cannot be said to be true or false.

Opposite numbers Two numbers whose sum is zero.

Order Numbers arranged in sequence from least to greatest, or vice versa, are in order.

Ordered pair A pair of numbers where order is important: (3,6) is not the same as (6,3).

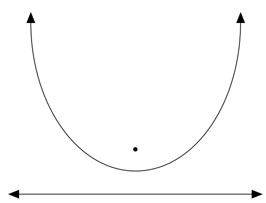
Order of symmetry The order of rotational symmetry is the number of ways a shape can be rotated to fit on itself. The number of axes of symmetry is the number of ways it can be reflected into itself. The total order of symmetry is the order of rotational symmetry plus the number of axes of symmetry.

Outcome The result of an experiment.

Oval
(a) An egg-shaped figure with 1 axis of symmetry.
(b) Another name for an ellipse.

Palindrome A word, sentence, or number that reads the same backwards or forwards, for example, madam, 19691.

Parabola A curve in which every point is the same distance from a particular point as from a particular line.



Parallel lines Two (or more) lines that have the same directions (if they are in the same plane then they are lines that never intersect).

Parallelogram A quadrilateral with opposite sides parallel (and congruent).

Pentagon A polygon with five sides.

Per cent A numeral for a fractional number whose denominator is 100, for example, 37 percent = 37% = $\frac{37}{100}$.

Percentage Another word for per cent.

Perimeter The boundary of a plane figure. Also the distance round this boundary.

Perpendicular bisector A line that bisects a line segment and is perpendicular to it.

Perpendicular lines Lines that intersect to form a right angle.

Pi (π) For a circle, a number that equals the length of the circumference divided by the length of the diameter. It is about 3.

Pictograph A graph drawn with pictures.

Pie graph (or pie chart) A circle graph cut into sectors.

Place value The value of the place a digit occupies, for example, in 57 the 5 occupies the tens place.

Plane A flat surface that extends indefinitely.

Point The intersection of two lines.

Polygon A plane figure bounded by the union of three or more line segments that form a simple closed curve. (Sometimes polygon refers only to the boundary.)

Polyhedron A solid bounded by polygons. (Sometimes polyhedron refers only to the bounding surface.)

Polyomino A plane shape made of squares of the same size, each square being connected to at least one other by a common edge. (Domino, tromino, tetromino, pentomino....)

Power A product shown by a base number and an exponent, for example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$ is the fourth power of 3. (The index is sometimes called the power.)

Prime A whole number greater than 1 with 2 factors, itself and 1.

Prime factorisation Rewriting a number as a product of primes.

Prism A polyhedron of uniform cross section with two congruent and parallel end faces.

Probability of an event A number that tells how likely it is that the event will happen.

Proportion An equation or equality between two ratios.

Protractor An instrument used to measure and draw angles.

Pyramid A polyhedron made up of a polygonal base and three or more triangular side faces which meet at a point (called the apex).

Quadrilateral A four-sided polygon.

Quotient The answer to a division problem. For example, 13 ÷ 3 = 4 with remainder 1, the quotient is 4; 13 ÷ 3 = $4\frac{1}{3}$, the quotient is $4\frac{1}{3}$.

Radius of a circle A line segment with one end point on the circumference and the other at the centre of the circle. It is also used for the length of radius.

Range The interval between the greatest and least values in a set of data.

Rate A fraction used to compare two quantities of different things, for example, 50 km/h or $6/m.

Ratio A fraction used to compare two quantities of the same things.

Rational number A number that can be written as a fraction or ratio.

Ray A part of a line that begins at a point and goes endlessly in one direction. The ray that begins at P and goes through *Q* is written $\overrightarrow{PQ}$ and *P* is its starting point.

Reciprocal The reciprocal of a number is the number obtained by interchanging the top line and the bottom line when the original number is written as a fraction, for example, 0.4 = $\frac{2}{5}$, reciprocal = $\frac{5}{2}$ = 2.5.

Rectangle A parallelogram with four right angles.

Rectilinear Straight-lined.

Reflection The movement of a figure when it is reflected (or flipped) over a line in the plane of the figure.

Reflex angle An angle with size between 180° and 360°.

Region The part of a plane inside a closed curve.

Regular polygon A polygon with all sides and angles congruent.

Regular polyhedron A polyhedron with all faces and angles congruent.

Relation A set of ordered pairs.

Remainder The amount left over after a division (21 ÷ 4 = 5 with remainder 1).

Replacement (probability) Recreating the conditions of an event so that the probability of any given outcome is the same on all trials, for example, the probability of drawing a queen from a deck of cards is the same on successive draws only if each card is replaced in the pack after it is drawn and inspected.

Rhombus A quadrilateral with four congruent sides.

Right angle An angle with size 90°.

Right-angled triangle A triangle that has one right angle.

Right prism A prism all of whose side faces are rectangles.

Right pyramid A pyramid whose apex is vertically above the mid-point of the base.

Rotation The movement of a figure when it is turned through an angle about a point in the plane.

Rotational symmetry A shape has rotational symmetry if it can be rotated on to itself.

Scale factor A number used as the factor for dilation, enlargement (or size transformation), and scale drawings.

Scalene triangle A triangle in which no sides are congruent.

Scatter diagram A diagram that shows pairs of variables on a graph to help statisticians see the connection between the two variables and see the spread of data.

Scatter plot Bivariate data represented as points on a co-ordinate graph. One variable is on the X-axis, the other on the Y-axis.

Second One second (written 1") equals $\frac{1}{60}$' or $\frac{1}{3600}$°.

Sector Part of a circle bounded by an arc and two radii.

Segment of a circle Part of a circle bounded by an arc and a chord.

Semicircle Half a circle bounded by a diameter and half a circumference.

Sequence An ordered set. (Usually an ordered set of numbers that forms a pattern according to a rule.)

Set A collection of things.

SI See Systéme Internationale d'Unites.

Similar Two figures are similar if they have exactly the same shape.

Size transformation Another name for a dilation (enlargement or reduction).

Simple closed curve A closed curve that does not cross itself.

Solution of a number sentence A number that makes the sentence true.

Solution set The set of all the solutions of an open sentence. (Also called the truth set.)

Sphere A solid bounded by a closed surface such that all points on the surface are a fixed distance from a given point.

Spherical triangle A triangle on the surface of a sphere formed by great circles of the sphere.

Square A square is a rectangle with congruent sides.

Square of a number A product in which the number is multiplied by itself, for example, the square of 3 is 3^2 = 3 x 3 = 9.

Square root of a number A number that when multiplied by itself produces the given number, for example, the square root of 9 is 3 or $^-3$. √9 means $^+3$.

Standard form The form in which a number is written as the product of a number between 1 and 10 and a power of 10, for example, 438.2 = 4.382 x 10^2. Note that calculators write this as 4.382 2. Another name for this is scientific notation.

Stem-and-leaf graph A bar graph made by arranging numerical data in a display, using the first part of the number as the stem and the last part as the leaf.

Data 16, 31, 25, 33, 27, 24, 14, 26, 31

1	4	6		
2	4	5	6	7
3	1	1	3	

Strip graph Represents frequencies as a proportion of a rectangular strip. For example:

Subset Set A is a subset of set B (written A $\subset$ B) if all the members of A are members of B.

Supplementary angles Two angles whose measures add to 180^o. One is called the supplement of the other.

Surface area The total outside area of the boundary of a solid.

Sum The result of adding two or more numbers.

Syllogism A group of three statements:

1 A general statement
2 A particular statement
3 A conclusion.

Symmetrical shape A shape is symmetrical if it has line symmetry or rotational symmetry, that is, if it can be reflected or rotated onto itself.

Systēme Internationale d'Unites (SI) A system of measuring units based on powers of 10 with basic units metre, kilogram, and seconds.

Tally chart A table with three columns headed "outcomes", "tally", and "frequency". For example:

Outcomes	Tally	Frequency
Shoes	|||	3
Sandals	~~||||~~		6
Gumboots	||	2
Running shoes	~~||||~~ |||	8

Term A member of a sequence or expression.

Tessellation A pattern of congruent shapes which cover a plane surface.

Tetrahedron A polyhedron with four triangular faces.

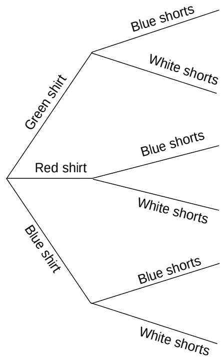
Total value of a digit The product of the place and face values, for example, 5 in 54 has total value fifty.

Translation The action of moving a figure so that all its points move the same distance in the same direction.

Transversal A line that intersects two (or more) other lines.

Trapezium A quadrilateral with a pair of parallel sides. (Trapezia: two or more of these.)

Tree Diagram A diagram which systematically represents all outcomes for a sequence of events. For example, if Ioane has 3 shirts and 2 pairs of shorts, the choice of outfits is:



Triangle A polygon with three sides.

Truncate Having the vertex (of a cone or pyramid) cut off by a plane surface (usually parallel to the base).

Two-by-two array A table of frequencies for data classified according to two factors each with two possible outcomes. For example:

	Fair hair	Dark hair
Left handed	1	2
Right handed	3	9

All values are listed in order.

Union The union of two sets A and B (written A $\cup$ B) is the set of all members of A or B or both.

Unit A particular quantity used as a standard of measurement.

Unit cost The cost of one unit of a particular item.

Universal set The set of all objects in the discussion.

Variables Quantities which can take many values, for example, the number of cars which may cross a bridge in the next hour, or the numbers which could replace the letter(s) (or pronumeral(s)) in an algebraic expression.

Vector An ordered set of numbers that can be used to describe a translation, for example, $\binom{3}{4}$.

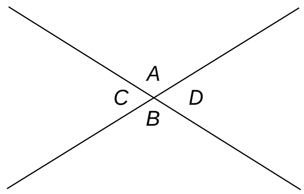
Velocity Speed in a particular direction.

Venn diagram A diagram used to show sets.

Vertex (vertices) A point where two or more rays or line segments meet. A corner point for a polygon or polyhedron.

Vertical line A line through the centre of the earth.

Vertically opposite angles Two angles on opposite sides of the vertex formed by 2 lines.



Thus A and B or C and D are vertically opposite.

Volume The amount of space occupied by an object. Volume is measured in cubic units.

Weight The measure of the heaviness of an object. (The word weight is often used, imprecisely, instead of mass.)

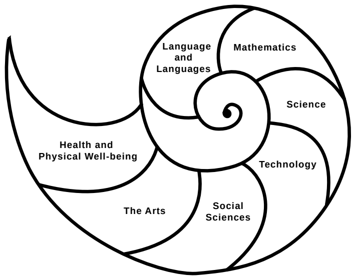
Whole number A number from the set {0, 1, 2, 3, ...}.

X-Axis The horizontal axis on a graph.

Y-Axis The vertical axis on a graph.

Zero The number 0 (nought).

Mathematics in the New Zealand Curriculum



Addendum to Level 8

Introductory Notes

Mathematics in the New Zealand Curriculum was published and released to schools in 1992. From 1995, mathematics programmes at all levels except form 7 must include the achievement objectives specified in the document. *Mathematics in the New Zealand Curriculum* will apply to all classes in mathematics from February 1996.

Since *Mathematics in the New Zealand Curriculum* was published there have been significant further developments which need to be taken account of in the mathematics education of senior students.

In light of these factors the Ministry of Education agreed to form a small working party to consider all of the above factors, and to draft an addendum to the national curriculum statement.

The working party also considered the need for schools to prepare students for a variety of unit standards leading to qualifications, other than pre-university requirements.

The working party recommended that a number of supplementary achievement objectives be added to *Mathematics in the New Zealand Curriculum*. These were published earlier this year in the form of a draft addendum and distributed to secondary schools, relevant tertiary institutions, and other interested persons for response.

Responses to the draft addendum indicated a high level of support for the inclusion of the achievement objectives listed. They also gave a number of suggestions for improved wording which have generally been incorporated. There was no call for significant change to the spirit and intention of the achievement objects.

Accordingly the achievement objectives, suggested learning experiences, sample assessment activities, and development band material listed below should now be read in conjunction with and become part of the national curriculum statement *Mathematics in the New Zealand Curriculum*.

The objectives are in three strands.

The Geometry Strand

Two achievement objectives are added covering the use of networks in critical path analysis. In *Mathematics in the New Zealand Curriculum,* networks are introduced at level 7, but this topic was not followed up at level 8. Research pointed to the need for this to be included.

A further achievement objective extends the co-ordinate geometry of level 7 to three dimensions. This continues the earlier emphasis on three-dimensional geometry.

The Algebra Strand

One new achievement objective and associated learning experiences relating to recurrence relations has been added.

Learning experiences and assessment examples to expand the scope of applications of the other algebra achievement objectives already in *Mathematics in the New Zealand Curriculum* are included.

The Statistics Strand

The additions reflect developments in the use of statistics in practice, particularly developments influenced by recent advances in spreadsheet software, statistical software, and the data-graphic features of other data-handling software. The new achievement objectives cover greater use of exploratory data analysis techniques, more on statistical aspects of experimental design, time-series investigations, and investigations using pairs of variables.

It is stressed that these additions do not require schools to amend programmes and do not imply additions to the Bursary prescriptions. National Qualification unit standards will be developed for all the objectives in the *Mathematics in the New Zealand Curriculum* national curriculum statement. Schools which wish to offer courses based on them may expect that qualifications credits will be available for successful students.

MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Geometry Level 8

Earlier levels in *Mathematics in the New Zealand Curriculum* place some emphasis on geometry in three dimensions. The objective below is designed to build on the co-ordinate geometry introduced at level 7 and use it to continue that emphasis.

Achievement Objective

Using co-ordinate geometry to explore shape and space

Within a range of meaningful contexts, students should be able to:

- describe and interpret points, directions, lengths, and planes in space using three-dimensional Cartesian co-ordinates.

Suggested Learning Experiences

Using co-ordinate geometry to explore shape and space

Students should be:

- using concrete models for plotting points and naming co-ordinates;
- using right-angled triangles in three dimensions and exploring ways of finding the distance between two points;
- plotting sets of points which satisfy a linear equation in three variables and verifying that the points lie on a plane.

Sample Assessment Activity

This assessment activity is an example of the kind of task which teachers could devise for their own assessment programme.

- Students investigate the values of variables that satisfy conditions of the type, $x < a$, $y < b$, $z < c$, and a general condition involving all three variables. This could be within the context of a linear programming problem, a physical problem in space, or some other application.

  Using this example, teachers could assess students' ability to:

  - describe and interpret points, directions, lengths, and planes in space using three-dimensional Cartesian co-ordinates (G8);
  - interpret information and results in context (MP8).

Sample Development Band Activities

- Students explore matrix transformations of the unit cube, using appropriate technology, as extensions of two-dimensional transformations.
- Students investigate the variety of shapes formed when a slice is made through a cube, or the variety of solids formed when similar pieces are removed from the corners of a cube.

The following achievement objectives allow the study of networks, started in level 7, to be continued.

Achievement Objectives

Exploring networks

Within a range of meaningful contexts, students should be able to:

- construct, use, and interpret network diagrams and find solutions to scheduling problems using the critical path algorithm;
- allocate resources using time charts.

Suggested Learning Experiences

Exploring networks

Students should be:

- constructing simple network diagrams (for example, for scheduling a construction project or replacing an organisation's computer network);
- investigating scheduling problems and network diagrams, including finding critical paths, and using the appropriate language (node, event, precedence dummy, critical path, slack (float) time, updating, time chart);
- using computer software to explore networks.

Sample Assessment Activity

This assessment activity is an example of the kind of task which teachers could devise for their own assessment programme.

- Students could construct a project network for the organisation of the exhibition described below which would include earliest and latest times of events and all listed tasks.

  An exhibition of precious artefacts is going from the Auckland City Art Gallery to a Sydney museum. The Australian Government takes responsibility for the artefacts from the closure of the Auckland exhibition to the closing of the Sydney exhibition. Students list the tasks to be performed and the time required for each. For example:

  > photographs taken for insurance and publicity purposes; insurance inspection; insurance policy issued; closing ceremony; packing; arrange contract for security from closing in Auckland to arrival in Sydney; contract security for Sydney; transport to Sydney; unpack and set up; advertising; compile souvenir booklet; print souvenir booklet.

  The project plan could be required to include: the critical path and duration of the project; an example of a dummy in a network and an explanation of its purpose; the effects on the project time and critical path caused by delays (such as a strike lasting four days, the booklet taking three extra days to produce).

Using this example, teachers could assess students' ability to:

- construct, use, and interpret network diagrams, and find solutions to scheduling problems using the critical path algorithm (G8);
- allocate resources using time charts (G8);
- devise and use problem-solving strategies to explore situations mathematically (MP8);
- interpret information and results in context (MP8);
- report the results of mathematical explorations concisely and coherently (MP8).

Sample Development Band Activities

- Students consider projects (such as building a bridge) where it is possible, by using more resources, to contract the duration of the tasks, assuming a linear relationship between cost and duration. They try to minimise the cost of reducing the duration. They recalculate the minimum time in which the project can be completed, and find the minimum cost for completion in that time.
- Students use networks to begin discussion of “if . . . then” relationships and construct truth tables for implication.

Additions to pages 164 to 167

Algebra Level 8

The following objective is designed to allow the development of courses in discrete mathematics.

Achievement Objective

Exploring patterns and relationships

Within a range of meaningful contexts students should be able to:

- use and interpret recurrence relations.

Suggested Learning Experiences

Exploring patterns and relationships

Students should be:

- interpreting problems that can be modelled by recurrence relations of the form $u_n = au_{n-1} + b$, and guessing and justifying general solutions to this type of relation;
- investigating sequences derived from the recurrence relation $u_n = au_{n-1} + bu_{n-2}$ (in particular the Fibonacci sequence for which $a = b = u_1 = u_2 = 1$) and investigating parallel situations such as those which arise when this condition does not hold;
- using computer software (particularly spreadsheets), or a programmable calculator to explore recurrence relations.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students investigate a situation such as the following. At the beginning of an experiment there are 1000 organisms present in a Petri dish. Unfortunately a drop of acid is dropped on the dish which kills 50 organisms every hour. Without the acid the number of organisms would double every hour. How many are present after two and after three hours?
  How many organisms are present after 10 hours?
  How long will it be before the organisms fill a dish of a given size?

  In a related experiment, the following numbers of organisms were counted at hourly intervals.

  (i)

Hour	1	2	3	4	5	6
Number	100	300	900	2700	8100	24300

  (ii)

Hour	1	2	3	4
Number	100	250	700	2050

  In each of the above experiments, determine the relation between organisms at time n and organisms at time n-1.
  Find and justify a general formula for the number of organisms after n hours.

Using this example, teachers could assess students' ability to:

- use and interpret recurrence relations (A8);
- devise and use problem-solving strategies to explore situations mathematically (MP8);
- prove or refute mathematical conjectures (MP8).

- Students generate the first 15 terms of the Fibonacci sequence.
They then investigate the truth or otherwise of relationships such as the following:
(F_n is $u_n = u_{n-1} + u_{n-2}$ with $u_1=u_2 = 1$)
$F_2 + F_4 + F_6 + . . . + F_{2n} = F_{2n+1} - 1$

(This could be proved by say (i) expanding F_{2r} using recurrence relations or (ii) mathematical induction.)

$F_1 + F_3 + F_5 + . . . + F_{2n-1} = F_{2n}$

$F_1 + F_4 + F_7 + ... + F_{3n-2} = (F_{3n})/2$

Using this example, teachers could assess students' ability to:

- use and interpret recurrence relations (A8);
- devise and use problem-solving strategies to explore situations mathematically (MP8);
- prove or refute mathematical conjectures (MP8).

Note: These are the kinds of results that students should generate for themselves. However, one or two of them can be used as an introduction to the topic.

Sample Development Band Activity

- Students investigate the solution of recurrence relations.

The learning experiences below relate to fitting functions or algebraic models to data as applications of the sixth objective on page 164 of Algebra in *Mathematics in the New Zealand Curriculum*. Work based on it could include the use of regression techniques from the statistics strand.

Suggested Learning Experience

Exploring patterns and relationships

Students should be:

- using graphs to explore existing data or data collected from experiments or investigations, investigating possible models to describe relationships, and developing methods for confirming the most appropriate model. This could involve the use of spreadsheets and/or other computer software.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students investigate a situation in which 150 red beans are mixed with 50 white beans and the mixture stirred. Twenty beans are removed and the red beans are replaced with white beans before returning them to the mixture. The process is repeated and a record is kept of the number of red beans in the mixture at each stage. Students find the relation between the number of red beans and the number of replacements made.

  Using this example, teachers could assess the students' ability to:

  - choose an appropriate model for real data, including the use of log-log and semi-log techniques, and analyse and interpret the results (A8);
  - find and use with justification, a mathematical model as a problem-solving strategy (MP8).

- Students find the relation between the time elapsed and the counter number on a tape recorder. They set the counter to zero at the beginning of the tape and record the counter number at five minute intervals.

  Using this example, teachers could assess students' ability to:

  - choose an appropriate model for real data, including the use of log-log and semi-log techniques, and analyse and interpret the results (A8);
  - use equipment appropriately when exploring mathematical ideas (MP8).

Sample Development Band Activity

- For a variety of sports records, students investigate the relationship between the record and its date to make predictions about when future improvements may occur. They could use a statistics package to do linear regression on transformed data and plot residuals to confirm the appropriateness of the model.

The Suggested Learning Experience below applies to the first two objectives of Exploring patterns and relationships, in the Algebra strand, on page 164 of *Mathematics in the New Zealand Curriculum*. It is intended to indicate possible applications to financial calculations that can be made using computer packages or programmable calculators.

Suggested Learning Experience

Exploring patterns and relationships

Students should be:

- using calculators and spreadsheets to explore the changing value of money over time, calculating present and future values of a cash flow, investigating the most desirable investment option, and applying compound interest and simple ordinary annuity models.

Sample Assessment Activity

- Students investigate a situation in which a person wishes to invest money to pay for future needs. It is estimated that she will need $50,000 in about 10 years' time. Prepare a report which investigates different investment options.

  Using this example, teachers could assess students' ability to:

  - use sequences and series to model real or simulated situations and interpret the findings (A8);
  - devise and use a mathematical model as a problem-solving technique (MP8).

Statistics — Level 8

The achievement objectives added to this strand are designed to further development in three areas:

- **the statistical aspects of experimental design**
- **time series**
- **the investigation of relationships in pairs of variables.**

Achievement Objectives

Statistical investigations

Within a range of meaningful contexts, students should be able to:

- use principles of experimental design to design an experiment, identifying aspects which must be considered, and conduct the experiment;
- use statistical methods and principles to analyse and report on experiments;
- use graphs, moving averages, separation into smooth and rough (with awareness of additive and multiplicative models) to explore time series;
- use indexes to deflate financial time series and to find inflation rate series;
- investigate relationships between two-category variables, using contingency tables, graphs and tables of percentages;
- investigate relationships between two continuous variables using graphs and model fitting methods including regression;
- use graphs and correlation co-efficients to assess the linearity of relationships and discuss the appropriateness of any regression line or correlation co-efficient.

Suggested Learning Experiences

Statistical investigations

Students should be:

- discussing approaches to experimentation using the language and terminology of experimental design (response and explanatory variables, control, placebo, single blind and double blind, replicate, treatment and level of treatment, randomisation, factor), considering ethical issues involved in experimentation;
- planning experiments; deciding on the questions to be investigated; planning data collection methods and undertaking pilot studies; exploring initial data through graphs; using the results of a pilot study to finalise the design of the experiment;
- summarising, presenting, and interpreting the results of an experiment using graphs, tables, and written descriptions (including discussing any assumptions that were made and reflecting on the purposes of the experiment and the extent to which they were met);
- graphing time series and describing their features;
- investigating the use of commonly used index numbers (for example, the CPI, the SE Forty, indices related to weather patterns such as the Southern Oscillation);

- discussing time-series data and using the appropriate language:
  - index, base, inflation, rate, price relative, weighted mean;
  - components of time series (smooth and rough, trend, cycles, seasonal effect, remainder), moving average, forecasts;
- constructing, using, and interpreting contingency tables from data-sets involving category variables;
- using scatter plots, and scatter plots with regression lines and other models drawn in to find the features of a relationship;
- using graphs to assess the appropriateness of linear regression and correlation;
- using software to calculate least squares regression equations and making predictions from them;
- exploring the meaning of a correlation co-efficient using graphs, for example, the strength of the relationship between people's golf handicaps and the length of time the game has been played by those people;
- discussing the connection between correlation and causality.

Sample Assessment Activities

These assessment activities are examples of the kinds of tasks which teachers could devise for their own assessment programme.

- Students write a report to analyse and evaluate aspects of the methodology of a published statistical experiment.

  Using this example, teachers could assess students' ability to:
  - use statistical methods and principles to analyse and report on experiments (S8);
  - report the results of mathematical explorations concisely and coherently (MP8).
- Students design and report on an experiment to determine if drinking a cup of coffee increases the pulse rate. This could involve prior research into how long it takes the coffee to affect the system. Factors could include milk, sugar, temperature, gender, prior consumption of coffee, age, or level of activity during the experiment. The need for controls should be considered, including the use of placebos (such as decaf). Ethical issues such as consent, confidentiality, and effects on the subject could be discussed.

  Using this example, teachers could assess students' ability to:
  - use principles of experimental design to design an experiment, identifying aspects which must be considered, and conduct the experiment (S8);
  - report the results of mathematical explorations concisely and coherently (MP8).
- Students obtain data relating to the gender and smoking habits of persons over 20 (whether or not the persons themselves classify themselves as smokers). They investigate whether there is evidence that smoking and gender are related.

  Using this example, teachers could assess students' ability to:
  - investigate relationships between two-category variables using contingency tables, graphs and tables of percentages (S8);
  - use tables of multi-variate data from social contexts to find the probabilities of everyday events or the proportion of outcomes in a given category (S6);
  - make conjectures in a mathematical context (MP8);
  - confirm or refute mathematical conjectures (MP8).

- Students study the typical price of petrol over (say) a ten year period and construct a price index series for petrol over this period. They decide if petrol is relatively better value for money at the end of the period compared with the beginning.

  Using this example, teachers could assess students' ability to:

  - use graphs, moving averages, separation into smooth and rough (with awareness of additive and multiplicative models) to explore time series (S8);
  - report the results of mathematical explorations concisely and coherently (MP8).

- Students use bivariate data, such as life expectancy and the number of doctors per capita for different countries, to calculate the correlation co-efficient between the variables and discuss relationships within the data.

  Using this example, teachers could assess students' ability to:

  - use graphs and correlation co-efficients to assess the linearity of relationships and discuss the appropriateness of any regression line or correlation co-efficient (S8).

Sample Development Band Activity

- Students investigate Simpson's Paradox, that is, the reversal of the direction of a comparison or association when data from several groups are combined to form a single group.

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