This example demonstrates how a teacher established the mathematics content for an integrated unit on the solar system and then differentiated the content to support all his year 9 students to explore some key mathematical ideas in Number. The focus was on whole numbers greater than 1,000,000, which students would need to be able to work with in problem solving in the unit. The example shows how the teacher used adaptations and differentiations to ensure all students participated and learned.

**Task**

When planning the integrated unit, the students’ subject teachers adopted a team approach and collaborated to ascertain the needs of the students. The unit theme was “New worlds, new beginnings” – the United Nations had decided to fund a settlement on another planet, and the students were required to present a plan for this. As part of their learning in the unit and in their final presentation, they were to demonstrate the planet’s characteristics and the size and scale of the new settlement.

**New Zealand Curriculum achievement objectives**

**Level 2**
- Know how many ones, tens, and hundreds are in whole numbers to at least 1000.
- Create and use appropriate units and devices to measure length, ... temperature, and time.

**Level 3**
- Know how many tenths, tens, hundreds, and thousands are in whole numbers.
- Use linear scales and whole numbers of metric units for length, ... temperature and time ...

**Level 4**
- Know the relative size and place value structure of positive and negative integers and decimals to three places.
- Use appropriate scales, devices, and metric units for length, ... temperature, ... and time.
- Convert between metric units, using whole numbers and commonly used decimals.

**Level 5**
- Know and apply standard form, significant figures, rounding, and decimal place value.
- Select and use appropriate metric units for length, ... temperature, ... and time, with awareness that measurements are approximate.
- Convert between metric units, using decimals.
Opportunities to use and develop key competencies

Students were thinking as they asked questions to clarify their understanding and as they justified their answers. They were relating to others when they worked together and shared their understanding. Students used language, symbols, and texts as they read out and recorded large numbers in a dice game using materials. Students were working in pairs or small groups, participating and contributing to complete a collaborative task.

Class description

Mr Babich teaches a year 9 mathematics class in an all-girls school with mixed ability classes in years 9 and 10. The students in the class were working in curriculum levels 2–5 for mathematics and statistics.

- **Aria** is a Māori student in her first year in an English-medium setting. Her attendance at kura during her primary years was irregular as a result of changing schools frequently. She has trouble reading and understanding English and Māori, and receives support for reading from a specialist literacy teacher. Aria is able to use Google Translate to translate unfamiliar English words into Māori. Her reading and writing ability generally allows her to work within level 3 of the New Zealand Curriculum. She is working at curriculum level 2 in mathematics, progressing towards level 3. She enjoys looking things up on electronic devices, including her phone. She is still developing peer relationships at school. A teacher’s aide attends some of Aria’s mathematics and English classes each week to provide her and her teachers with additional support.

- Three students (**Maria**, **Cala**, and **Tia**) are working at level 2–3 in mathematics. While lacking confidence in their work and having low expectations of themselves as mathematicians, they are capable of achieving in level 3 and working towards level 4. The mathematics focus for these students is to develop number knowledge and skills and to build their ability to apply this learning within diverse mathematics contexts. The teacher is working on developing their confidence to take risks in a safe environment and to see the relevance of mathematics in their lives.

- **Lien** is an English language learner (ELL) who recently arrived from China and is becoming familiar with a new culture and way of learning. She usually sits by herself and tends not to interact with her peers due to a lack of confidence and limited English. She is attending three ELL classes a week. Lien is able to work with numbers within equations, but when the problems are put into context, English is a barrier and she is unable to interpret and therefore solve the problems. Within the New Zealand Curriculum her mathematical knowledge is at level 5, but her literacy ability only allows her to work within level 2.

Teaching as inquiry

At the start of this project, Mr Babich led a discussion with the class about settling a planet to ascertain what mathematical ideas would support students’ learning in this context. He and the students made a list of content that could be included in their project, and he identified skills gaps as well as connections with other learning areas (such as science). For this series of lessons, he was focusing on supporting all students to access key mathematical ideas about place value and to use decimals and whole numbers into the millions to solve problems involving distance and time. (In the project, students would be looking at the distance to other planets and the time needed to get there.)
In this class, students’ understandings of Number ranged from level 2 to 5. Mr Babich conferred with the English teacher to ascertain the students’ literacy levels so that he could support them in both accessing information and articulating and recording their mathematical understanding. He represented content in multiple ways and worked with students with additional learning needs one-on-one and in small groups to reinforce mathematical ideas and practise the required skills.

**Focusing inquiry**
*What was important (and therefore worth spending time on), given where Mr Babich’s students were at?*

Mr Babich had previously gathered information through formative assessment, an end-of-term test, conversations with students, and students’ notes in notebooks and on electronic devices. He had identified that some of the students showed a lack of understanding of whole numbers into the thousands (which relates to achievement objectives at level 3). Both Mr Babich and the science teacher, Mr Sims, identified that this was mathematics knowledge that all the students needed at the start of the project.

Mr Babich wanted to focus on moving the students towards recognising, understanding, and using large whole numbers. He collaborated with Mr Sims so that students’ understanding of these numbers would be reinforced in a science context. He also wanted to build on students’ prior knowledge about decimals, which they had learned about in the previous term.

**Teaching inquiry**
*What teaching strategies (evidence-based) helped Mr Babich’s students learn?*

**Learning inquiry**
*What happened as a result of the teaching, and what were the implications for future teaching?*

**Scaffolding to support understanding**
- As a starter activity, Mr Babich had an enlarged A3 ‘place value houses’ diagram on the whiteboard. Students took turns reading numbers aloud while referring to the diagram on the whiteboard. He put the word ‘and’ underneath the relevant places on the diagram as a scaffold to support students to say numbers correctly.

- The students then played a dice game in pairs, using a laminated copy of the place value houses diagram and a 0–9 dice. The aim was to create the biggest number possible from their dice throws. They then formed into groups of three or four pairs. As a group activity, they looked at each pairs’ numbers and read them all out before ordering them from biggest to smallest.

- Using Quizlet, Mr Babich had developed sets of flashcards for matching numbers expressed as numerals and as words: one set to 10 000, one to 1 000 000, and another over 1 000 000. While the class were doing their pair activities, he worked with Aria on a memory game using the flashcards to 10 000.

**Mr Babich:** I noticed that students were reading numbers as a list of digits and not correctly saying them as a written numeral. Once I started using the word ‘and’ as a visual prompt, they became much more proficient at this task.

**Cala:** I have learnt how to say big numbers before, but I can’t remember which number goes in which place, so seeing the diagram really helps.

**Mr Babich:** I knew that Aria had difficulty reading numbers to a million, given their many zeros and place holders. I realised that she needed more intensive work on this, so developed the flashcards to give her practice.
### Teaching inquiry
**What teaching strategies (evidence-based) helped Mr Babich’s students learn?**

- While Mr Babich worked with Aria, the teacher’s aide worked with Lien on the same activity using the flashcards over 1,000,000. This was to give Lien extra practice reading English words. She was given a diagram of the place value houses with the English words to help her connect the words to her mathematics knowledge.
- As a follow-up, Mr Babich asked two students to partner up with Aria and Lien to play the memory game with the flashcards.

### Learning inquiry
**What happened as a result of the teaching, and what were the implications for future teaching?**

- Mr Babich: The numeral system in China is different, but Lien has picked up the English system of place values by using the diagram.
- Miss Tupelo (teacher’s aide): Lien used to say the number first in Cantonese, then English. By the end of the activity she was saying the numbers in English first.
- Aria: I’ve got this. Let’s move to the next set! I want to work on numbers with seven digits with lots of zeros.

### Making mathematical concepts visible and tangible

- Mr Babich realised that some students needed to understand how to multiply and divide by a power of 10. He designed an activity to target Maria, Cala, and Tia, using a physical representation of the process. He prepared cards showing the numbers 0–9, a multiplication and a division sign, and a decimal point.
- He recapped with the class how to multiply and divide by 10, 100, and 1000. Volunteers then lined up with the cards in the front of the classroom. The student holding the decimal point was asked to multiply or divide by 10, 100, or 1000. She stayed in the same place while instructing the others to move around her in keeping with the operation they were enacting.

### Making connections to the context

- Mr Babich had a discussion with the class about the solar system and showed them a website that had an interactive tour function. He encouraged students to use their electronic devices to explore the website further and look up the sizes of different planets and their distance from Earth. He asked them to work individually to choose three planets, find the distance from Earth for each, then order and say the distances.
- While most of the class worked independently on this task (asking the teacher’s aide if they had a question), Mr Babich worked with Aria, Lien, Maria, Cala, and Tia. They took turns reading the numbers out to each other and writing them as words and as numerals.

### With your colleagues, discuss:
- What do you need to do to make mathematics content and key ideas accessible to all your learners?

The teacher used a whole-class activity to illustrate place value and connect with students’ prior knowledge of decimals.

### With your colleagues, discuss:
- What can you do to make mathematical concepts more tangible for all your students?

The teacher applied the recent learning within the wider context of the unit.

### Mr Babich: I believe this really helped the girls’ understanding. It was great to see Tia, who barely puts up her hand normally, raise her hand and correctly answer my question.

Tia: I really have got this. It’s not the decimal point moving - the numbers are ten times smaller or ten times bigger.

### Mr Babich: I believe the concrete and physical nature of this activity played a big part in helping the girls understand how to multiply and divide by a power of ten. I need to find different ways of presenting other mathematical concepts when students don’t appear to be making sense of my teaching.

### Mr Babich: I noticed in both the small group and the pairs that some students didn’t have any concept of how long a kilometre is. I’m going to work on this with the class tomorrow by having them walk a kilometre and then estimate how far from the school their homes are. I’ll get them onto Google Maps to check the accuracy of their estimates.
<table>
<thead>
<tr>
<th>Teaching inquiry</th>
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<td>• The whole class then paired up to compare their numbers and find the difference between their smallest and largest distance using a subtraction calculation.</td>
<td>Mr Babich: Lien was extremely quick at subtraction with the big numbers. Maria, Cala and Tia found this task hard, so I need to address this in my next unit with this class.</td>
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<td>• Mr Babich roved the classroom to support students with their calculations and to take note of the strategies they were using to guide future teaching.</td>
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<td><strong>Transferring the learning</strong></td>
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<td>• Mr Sims (science teacher) planned a series of activities in subsequent science lessons to consolidate the students’ learning.</td>
<td>Mr Sims: I did the solar system activity straight after the students had been finding out distances between planets in maths. We used a basketball to represent the sun and found objects (pea-sized things, ball bearings, etc.) that were the approximate sizes of the planets relative to the sun. We measured out the distance between planets at the same scale. We could only fit the first four planets on the field, then we had to start wrapping around ... Once the students had walked a kilometer in maths, Maria, Cala, and Tia were able to estimate that the outermost planets would be in the next suburb!</td>
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<td>• In the main activity, he provided balls and other round objects for the class to use to form a representation of the solar system. The students wrote on strips of paper the distances between Earth and other planets that they had identified in their mathematics class. They then developed a scale model using the round objects and strips of paper attached to string.</td>
<td>Maria: I can see how what I learnt in maths I can use in science!</td>
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<td>• Mr Sims finished the series of lessons by talking about how fast light is, how long it takes light to travel from the sun to Earth, and the idea of a light year.</td>
<td>Mr Sims: Usually when I teach a year 9 class they find it difficult using large numbers. But it’s different this time. The kids were not afraid of them and could move on to the science concepts. Though I noticed that Maria and Cala need further work on scale. They were fine dividing by ten but need more practice with larger numbers. I’ll feed this back to Mr Babich, as I know he’s working on this in maths.</td>
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<td></td>
<td>Mr Sims: Because of the joint focus in maths and science I’m sure this class understood the scale in these concepts much better than classes in previous years.</td>
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### Teaching inquiry
*What teaching strategies (evidence-based) helped Mr Babich’s students learn?*

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<td>• Both teachers used formative assessment, noting who could read the numbers, who could multiply by 10, 100, and so on, and who could complete the calculation between the biggest and smallest distance. Mr Sims also made observations about students’ ability with scale for the anecdotal notes on individual achievement.</td>
<td>Lien: <em>I couldn’t read numbers before, but now I know I have to say ‘and’ in the right place. I’m not scared to read out big numbers in class anymore.</em> Mr Babich: <em>I noticed that Lien is making really quick progress. I am going to make a time with her to talk about what English support she needs next in maths.</em></td>
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<td>• For self-assessment, the students filled in a profile sheet that included assessing their reading and understanding of numbers into the millions against achievement levels defined on the sheet.</td>
<td>The students completed a self-assessment sheet.</td>
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### Next steps

Now that you have explored this example, work with colleagues to:

- consider the challenges and opportunities in relation to inclusion for your students
- decide on the next steps in your mathematics teaching to ensure all your learners are participating, learning, and achieving
- plan for a future meeting to review the impact of your next steps and what now needs to happen.

### Recommended resources

- **NOVA website** has information about the solar system with an interactive tour function.